Nonlinear Analysis of Fiber-Reinforced Composite Laminates Subjected to Uniaxial Tensile Load

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ABSTRACT: A nonlinear constitutive model together with a mixed failure criterion for a single lamina is developed to simulate the behavior of composite laminates under uniaxial tension. In the model, fiber and matrix are assumed to behave elastic–plastic and the in-plane shear to behave nonlinear with a variable shear parameter. The damage onset for individual lamina is detected by a mixed failure criterion, which is composed of Tsai–Wu criterion and maximum stress criterion. After damage is taken place within the lamina, fiber and in-plane shear are assumed to exhibit brittle behavior and matrix to exhibit degrading behavior. This material model has been tested against experimental data and good agreement has been obtained.

KEY WORDS: constitutive model, elastic–plastic, nonlinear, shear parameter, mixed failure criterion, post-damage mode.

INTRODUCTION

DUE TO LIGHTWEIGHT and high strength, the use of fiber-reinforced composite laminate materials in aerospace industry or in applied engineering has increased rapidly in recent years. In numerous cases involving the design of composite structures, there is a need for more refined analysis that takes into account phenomena such as progressive cracking and inelastic or nonlinear deformation of the composite materials.

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Such analysis is required not only to predict the deformational response, but also to provide a method to evaluate the accurate stresses to be used in failure predictions.

Most of the advanced composite materials have organic matrices; therefore, there is a significant nonlinear stress–strain behavior present in the transverse direction of lamina and particularly in shear deformation [1]. A significant number of macro-mechanical models have been proposed to represent the constitutive relation of fiber-reinforced composite materials such as nonlinear elasticity models [2,3], plasticity models [4–8], or damage theory coupled with elasticity [9]. In addition, various failure criteria have also been proposed to predict the onset of damage in single layer within the fiber-reinforced composites. There are four types of failure criteria: (a) limit theories, (b) polynomial theories, (c) strain energy theories, and (d) direct mode determining theories. The limit theories compare the value of each stress or strain component to a corresponding ultimate value, such as maximum stress theory and maximum strain theory [10]. The polynomial theories use a polynomial in stress to describe a failure surface, such as Tsai–Wu failure criterion [11] and Hoffman failure criterion [12]. The strain energy theories attempt to use a nonlinear energy based criterion to define failure, such as Tsai–Hill failure criterion [13]. Finally, the direct mode determining theories are usually with polynomials in stress and use separate equations to describe each mode of failure, such as Hashin failure criterion [14], Lee failure criterion [15] and Chang failure criterion [16]. As for the post-damage process of individual lamina, there are two idealized types of failure modes defined in the previous study [5]; namely, brittle and ductile. For the brittle mode, the material is assumed to give up its entire stiffness and strength in the dominant stress direction as the damage is reached, whereas for the ductile mode the material remains its strength but loses its overall stiffness in the damage direction.

Obviously, a completely and rationally mechanical response analysis of individual layer within the laminate under loading must be included three parts; namely, pre-damage analysis, damage onset determining, and post-damage analysis. In the pre-damage analysis the proper constitutive model of lamina is a key tool to describe the real behavior of individual layer within the laminate under loading. In the previous study, it is assumed that the fiber and matrix perform as elastic–plastic behavior [5] and the in-plane shear behaves nonlinear with a constant shear parameter [16]. In this study, however, it is proposed that the in-plane shear behaves nonlinearly with a variable shear parameter. The difference between these two distinct types of shear parameter is investigated in this literature. In the past, the Tsai–Wu failure criterion is the most common criterion used to determine the damage onset of individual layer. However, Zhu and Sankar [18] proposed that the combination of both Tsai–Wu and maximum stress criteria was a much better criterion for damage determining of lamina. Thus, in this paper the so-called mixed criterion, a combination of Tsai–Wu criterion and maximum stress criterion, is employed to determine the damage onset of individual layer within the laminate under loading. For the post-damage analysis, a degrading mode for matrix and brittle modes for fiber and in-plane shear are proposed to simulate the post-damage behaviors of individual lamina.

In this paper, a proposed nonlinear analysis model included various post-damage modes is described first. Second, a material constitutive model considering the nonlinear in-plane shear behavior with variable shear parameter and the elastic–plastic behavior of fiber and matrix is developed. Third, various failure criteria and post-damage modes are reviewed, and a mixed failure criterion and the post-damage modes are proposed. Fourth, the laminate governing equations are built up to describe the incremental force–strain relations of the composite laminates. Then, the ABAQUS finite element program is used to carry out numerical analyses for laminates with various configurations and various off-axis loads. The results predicted by different failure criteria and post-damage modes are compared with each other. Finally, numerical results for the proposed nonlinear analysis model are compared with the work done by Vaziri et al. [5] and against the experimental data by Petit and Waddoups [2].

NONLINEAR ANALYSIS MODEL

Idealized Stress–Strain Curve and Post-Damage Model

For a single lamina under loading, the stress–strain curves of the proposed nonlinear analysis model are shown in Figure 1. The model comprises three basic regions in fiber and matrix; namely, the elastic, plastic, and post-damage regions. And, it comprises two basic regions in shear; namely, the nonlinear, and post-damage regions. For the present model, it is assumed that the material response can be adequately represented by bilinear stress–strain curves in the principal material directions, 1-direction (fiber direction) and 2-direction (transverse direction) of the lamina, and by a nonlinear stress–strain curve for in-plane shear in 1-2 direction. During the nonlinear stage in shear direction, the nonlinear shear modulus, G_{12n}, is depending on the shear strain, γ_{12}. In the pre-damage regions, the elastic modulus for elastic stage is denoted by E_{iie} $(i=1,2)$, and the elastic modulus for plastic stage is denoted by E_{iip} $(i=1,2)$, in the principal material directions, respectively. In the post-damage region, the elastic stiffnesses are dropped to zero (brittle modes) in 1-direction and 1-2 direction. However, the elastic stiffness is assumed to have a negative modulus, E_{22f} (degrading mode) in 2-direction. This means that the damaged lamina unloads in the transverse direction through a negative tangent modulus until no load remains in the lamina.

Nonlinear Constitutive Model of the Lamina

For fiber-composite laminate materials, each lamina can be considered as an orthotropic layer in a plane stress condition. Taking into account the elastic–plastic behaviors in the 1-direction and 2-direction and the nonlinear behavior on the 1-2 plane within the lamina, the stress–strain relations for an orthotropic lamina in the material coordinates $(1,2)$ can be written as [3]

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_{11}} & \dfrac{-\nu_{21}}{E_{22}} & 0 \\ \dfrac{-\nu_{12}}{E_{11}} & \dfrac{1}{E_{22}} & 0 \\ 0 & 0 & \dfrac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} + S_{6666}\tau_{12}^2 \begin{Bmatrix} 0 \\ 0 \\ \tau_{12} \end{Bmatrix} \tag{1}$$

where ε_1, ε_2, and γ_{12} represent the strains in 1-direction, 2-direction and 1-2 plane, respectively. σ_1, σ_2 and τ_{12} denote the stresses in 1-direction, 2-direction and 1-2 plane, respectively. The terms ν_{12} and ν_{21} are the Poisson's ratios. The terms E_{11} and E_{22} are the elastic moduli in 1-direction and 2-direction. If the material in 1-direction or 2-direction is

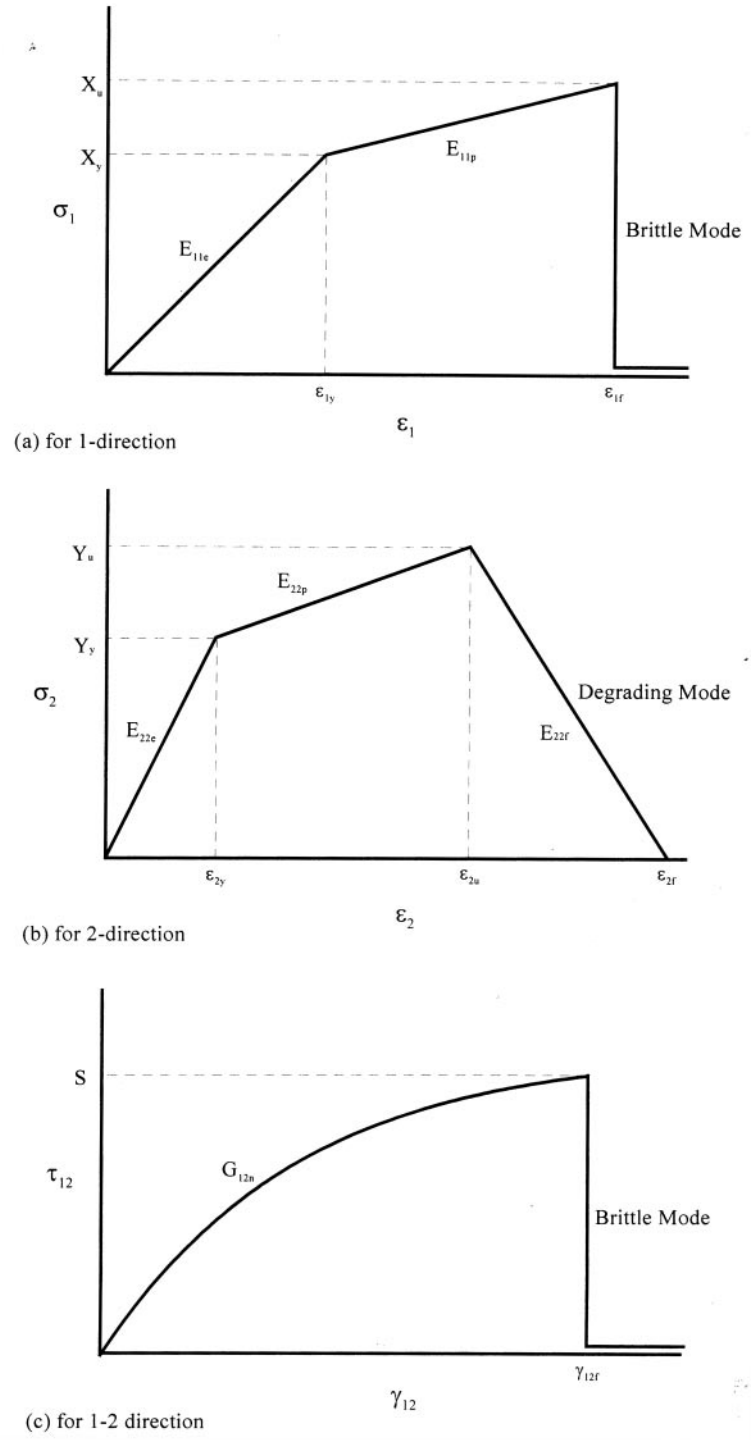


Figure 1. *Stress–strain curves of the proposed nonlinear failure model.*

in the elastic stage, then $E_{11} = E_{11e}$ or $E_{22} = E_{22e}$. And if the material is in the plastic stage in 1-direction or 2-direction, then $E_{11} = E_{11p}$ or $E_{22} = E_{22p}$. The G_{12} is the shear modulus and S_{6666} is a shear parameter to account for the in-plane shear nonlinearity. The S_{6666} is a function of shear strain and can be determined by fitting the stress–strain curve of pure shear test data.

The incremental stress–strain relations for a nonlinear orthotropic lamina can be given as follows:

$$\Delta\{\sigma'\} = [Q'_1]\Delta\{\varepsilon'\} \tag{2}$$

$$\Delta\{\tau'_t\} = [Q'_2]\Delta\{\gamma'_t\} \tag{3}$$

where $\Delta\{\sigma'\} = \Delta\{\sigma_1, \sigma_2, \tau_{12}\}^T$, $\Delta\{\tau'_t\} = \Delta\{\tau_{13}, \tau_{23}\}^T$, $\Delta\{\varepsilon'\} = \Delta\{\varepsilon_1, \varepsilon_2, \gamma_{12}\}^T$, $\Delta\{\gamma'_t\} = \Delta\{\gamma_{13}, \gamma_{23}\}^T$, and

$$[Q'_1] = \begin{bmatrix} \dfrac{E_{11}}{1-\nu_{12}\nu_{21}} & \dfrac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ \dfrac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} & \dfrac{E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & \dfrac{1}{1/G_{12} + 3S_{6666}\tau_{12}^2} \end{bmatrix} \tag{4}$$

$$[Q'_2] = \begin{bmatrix} \alpha_1 G_{13} & 0 \\ 0 & \alpha_2 G_{23} \end{bmatrix} \tag{5}$$

The terms α_1 and α_2 are the shear correction factors and are taken to be 0.83 in this study [17]. It is assumed that the transverse shear stresses always behave linearly and do not affect the nonlinear in-plane behavior of individual lamina.

FAILURE CRITERION AND DEGRADATION OF STIFFNESS

Review of Failure Criteria

As previously mentioned, failure criteria fall into four basic categories: (1) limit theories, (2) polynomial theories, (3) strain energy theories, and (4) direct mode determining theories. Among them, three types of failure criteria, i.e., maximum stress criterion, Tsai–Wu failure criterion and Chang failure criterion, are selected to be reviewed and numerical results based on these failure criteria are compared with each other.

MAXIMUM STRESS CRITERION

The maximum stress criterion is the dominant member of the limit failure theory category. For the plane stress condition, the maximum stress criterion for an orthotropic material can be expressed as follows:

$$\frac{\sigma_1}{X_{ut}} = 1 \quad \text{or} \quad \frac{\sigma_1}{X_{uc}} = 1 \tag{6}$$

$$\frac{\sigma_2}{Y_{ut}} = 1 \quad \text{or} \quad \frac{\sigma_2}{Y_{uc}} = 1 \tag{7}$$

$$\frac{\tau_{12}}{S} = 1 \tag{8}$$

where X_{ut}, Y_{ut} and X_{uc}, Y_{uc} are the ultimate longitudinal strengths and ultimate transverse strengths in tension and compression of the lamina, and S is the ultimate in-plane shear strength.

TSAI–WU FAILURE CRITERION

The Tsai–Wu failure criterion has a general nature, because it contains almost all other polynomial theories as special cases. Under the plane stress condition, the Tsai–Wu failure criterion has the following form:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 = 1 \tag{9}$$

where

$$F_1 = \frac{1}{X_{ut}} + \frac{1}{X_{uc}}, \quad F_{11} = \frac{1}{X_{ut}X_{uc}}, \quad F_2 = \frac{1}{Y_{ut}} + \frac{1}{Y_{uc}}, \quad F_{22} = \frac{1}{Y_{ut}Y_{uc}}, \quad F_{66} = \frac{1}{S^2}.$$

The stress interaction term F_{12} in Equation (9) is difficult to be determined and Narayanaswami and Adelman [19] suggested that F_{12} could be set equal to zero for practical engineering applications. Therefore, $F_{12} = 0$ is used in this study.

CHANG FAILURE CRITERION

As previous described, some direct mode determining failure criteria have been proposed, such as Hashin failure criterion [14], Lee failure criterion [15], and Chang failure criterion [16]. These failure criteria provide separate failure equations for each mode of failure. In this paper, only the Chang failure criterion is reviewed. In a plane stress space, Chang uses five distinct polynomials to describe five modes of failure, which are discussed below:

1. Fiber breakage mode

$$\frac{\sigma_1}{X_{ut}} = 1 \tag{10}$$

2. Fiber buckling failure mode

$$\frac{\sigma_1}{X_{uc}} = 1 \tag{11}$$

3. Matrix tensile cracking mode

$$\left(\frac{\sigma_2}{Y_{ut}}\right)^2 + \frac{(\tau_{12}/2G_{12}) + (3/4)S_{6666}\tau_{12}^4}{(S^2/2G_{12}) + (3/4)S_{6666}S^4} = 1 \tag{12}$$

4. Matrix compression failure mode

$$\left(\frac{\sigma_2}{Y_{uc}}\right)^2 + \frac{(\tau_{12}/2G_{12}) + (3/4)S_{6666}\tau_{12}^4}{(S^2/2G_{12}) + (3/4)S_{6666}S^4} = 1 \tag{13}$$

5. Fiber–matrix shearing failure mode

$$\left(\frac{\sigma_1}{X_{ut}}\right)^2 + \frac{(\tau_{12}/2G_{12}) + (3/4)S_{6666}\tau_{12}^4}{(S^2/2G_{12}) + (3/4)S_{6666}S^4} = 1 \tag{14}$$

or

$$\left(\frac{\sigma_1}{X_{uc}}\right)^2 + \frac{(\tau_{12}/2G_{12}) + (3/4)S_{6666}\tau_{12}^4}{(S^2/2G_{12}) + (3/4)S_{6666}S^4} = 1 \tag{15}$$

It should be noted that S_{6666} is a constant in Chang failure criterion.

Proposed Mixed Failure Criterion

Although the Tsai–Wu failure criterion is widely used in determining the damage onset of a lamina, there are some drawbacks with it. Among them is the fact that the failure stress of fiber in a lamina exceeds the strength of material for the case of symmetric angle-ply laminates with small fiber angle (say $0° < \theta < 20°$) subjected to off-axis tension. In order to eliminate this unreasonable phenomenon, the limitation of maximum stress of fiber is added into the Tsai–Wu failure criterion to obtain a mixed failure criterion. For the plane stress condition, neglecting the stress interaction term F_{12} in Equation (9), the mixed failure criterion can be written as the following formulations:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 = 1 \tag{16}$$

and

$$\sigma_1/X_{ut} \leq 1 \quad \text{or} \quad \sigma_1/X_{uc} \leq 1 \tag{17}$$

Normalized Failure Stresses and Failure Contribution

The Tsai–Wu failure criterion and the proposed mixed failure criterion consider the coupling effect of in-plane stresses, σ_1, σ_2 and τ_{12}, in the lamina when the collapse occurs. In order to figure out the individual stress ratio and failure contribution in the lamina, two terminologies are defined, which are normalized failure stresses and failure contributions. The normalized failure stresses represent the stress ratios (failure stresses/corresponding strengths) in the lamina for various stresses at the onset of collapse, which are described as follows:

$$(\sigma_{11f})_n = \frac{\sigma_{11f}}{X_{ut}} \quad \text{or} \quad (\sigma_{11f})_n = \frac{\sigma_{11f}}{|X_{uc}|} \tag{18a}$$

$$(\sigma_{22f})_n = \frac{\sigma_{22f}}{Y_{ut}} \quad \text{or} \quad (\sigma_{22f})_n = \frac{\sigma_{22f}}{|Y_{uc}|} \tag{18b}$$

$$(\tau_{12f})_n = \left|\frac{\tau_{12f}}{S}\right| \tag{18c}$$

where $(\sigma_{11f})_n$, $(\sigma_{22f})_n$ and $(\tau_{12f})_n$ denote the normalized failure stresses in 1-direction, 2-direction, and 1-2 plane of the lamina, respectively. The σ_{11f}, σ_{22f} and τ_{12f} are the stresses of the lamina in 1-direction, 2-direction and 1-2 plane, respectively, at the onset of failure.

The failure contributions are defined as:

$$FC_{11} = F_1\sigma_1 + F_{11}\sigma_1^2 \tag{19a}$$

$$FC_{22} = F_2\sigma_2 + F_{22}\sigma_2^2 \tag{19b}$$

$$FC_{12} = F_{66}\tau_{12}^2 \tag{19c}$$

where FC_{11}, FC_{22} and FC_{12} indicate the failure contributions of σ_1, σ_2 and τ_{12}, respectively, when the lamina collapses.

Property Degradation Models

Upon damage within the lamina occurring, the material begins to degrade its property. Material degradation within the damaged area was evaluated based on the mode of failure predicted by the failure criterion. Therefore, the residual stiffnesses of composites strongly depend on the mode of failure in each layer. According to the literature, the property degradation models for each layer can be separated into three idealized types of failure modes named as brittle, ductile [5] and degrading mode [2]. For the brittle mode, the material is assumed to lose its entire stiffness and strength in the dominant stress direction, whereas for the ductile mode the material retains its strength but loses all of its stiffness in the failure direction. For the degrading mode the material is assumed to lose its stiffness and strength in the failure direction gradually until the stress in that direction is reduced to zero.

For the maximum stress theory, the stresses in principal material directions must be less than the respective strengths, otherwise fracture is said to have occurred. Although this failure criterion can distinguish the failure modes of material, the failure modes are all brittle types. For the Tsai–Wu failure theory, it cannot distinguish the failure modes and does not consider the post failure condition.

CHANG'S PROPERTY DEGRADATION MODEL

For the Chang failure theory, it not only can distinguish the failure modes but also consider the post failure conditions. The Chang's property degradation models [16] for each lamina are described as follows:

1. For the matrix tensile or compressive failure mode, the in-plane properties in the failed layer are reduced as

$$[Q_1'] = \begin{bmatrix} E_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{1/G_{12} + 3S_{6666}\tau_{12}^2} \end{bmatrix} \tag{20}$$

2. For the fiber breakage or buckling failure mode, the material in that region cannot sustain furthermore load. Thus the material properties for the failed layer and all other layers are reduced to zero.

$$[Q_1'] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{21}$$

3. For the fiber–matrix shearing failure mode, the material can still carry load in the fiber direction and in the matrix direction, but shear loads can no longer be carried. This is modeled by reducing the shear property and the Poisson's ratios, ν_{12} and ν_{21}, to zero.

$$[Q_1'] = \begin{bmatrix} E_{11} & 0 & 0 \\ 0 & E_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{22}$$

PROPOSED PROPERTY DEGRADATION MODEL

In this investigation, it is proposed that the post damage mode are idealized as the brittle behavior for σ_1 and τ_{12} and the degrading behavior for σ_2. The following three rules are used to determine whether the ply failure is caused by matrix fracture, shear failure, or fiber breakage or buckling [20]:

1. If a ply fails in the condition of $X_{uc} < \sigma_1 < X_{ut}$ and $-S < \tau_{12} < S$, the damage is assumed to be matrix induced. Consequently, the degradation of transverse stiffness occurs. Due to the interlock action with the neighboring plies, the damaged ply gradually loses its capability to support transverse stress, until the fracture in shear or the breakage or buckling in fiber on the same ply. But, the lamina remains to carry the longitudinal and shear stresses. In this case, the constitutive matrix of the lamina becomes

$$[Q_1'] = \begin{bmatrix} E_{11} & 0 & 0 \\ 0 & E_{22f} & 0 \\ 0 & 0 & \dfrac{1}{1/G_{12} + 3S_{6666}\tau_{12}^2} \end{bmatrix} \tag{23}$$

   where E_{22f} is a negative tangent modulus in transverse direction of lamina after matrix damage. In the proposed model, the shear parameter S_{6666} has variable value.
2. If the ply fails in the condition of $X_{uc} < \sigma_1 < X_{ut}$, and $\tau_{12} \geq S$ or $\tau_{12} \leq -S$, the damage is assumed to be shear induced. Consequently, the damaged lamina loses its capability to support transverse and shear stresses, but remains to carry longitudinal stress. In this case, the constitutive matrix of the lamina becomes

$$[Q_1'] = \begin{bmatrix} E_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{24}$$

3. If the ply fails with $\sigma_1 \geq X_{ut}$ or $\sigma_1 \leq X_{uc}$, the ply failure is caused by the fiber breakage or buckling and a total ply rupture is assumed. Thus, the constitutive matrix of the lamina becomes

$$[Q'_1] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{25}$$

LAMINATE GOVERNING EQUATIONS

The forgoing nonlinear failure analysis model for fiber-reinforced composite lamina can be combined with classical lamination theory to form the following incremental laminate force–strain relations:

$$\Delta\{N\} = \sum_{i=1}^{n} [Q]_i t_i \Delta\{\varepsilon\} \tag{26}$$

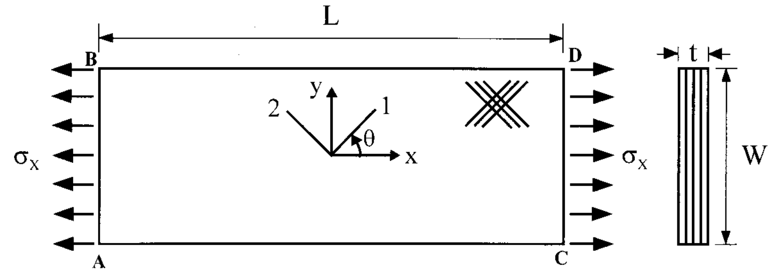
where $\Delta\{N\} = \Delta\{N_x, N_y, N_{xy}\}^T$ and $\Delta\{\varepsilon\}$ are the vectors of the resultant membrane forces and the incremental strains in the overall laminate coordinate system (x, y), respectively. The term t_i is the thickness of the ith layer, n is the number of layers. The matrix $[Q]_i$ stands for constitutive matrix for the ith layer and can be obtained by proper rotation of the $[Q'_1]$ matrix of that layer.

NUMERICAL ANALYSIS

Numerical Calculation and Material Properties

The aforementioned nonlinear constitutive model combined with various failure criteria and various post damage modes for composite materials are implemented into a FORTRAN subroutine and linked to the ABAQUS finite element program [21]. The analyzed laminates are simply supported around all edges of the plate as shown in Figure 2. The ply orientation of the laminate can be selected arbitrarily, but it must be symmetric with respect to the middle plane of the plate. The laminate is subjected to uniaxial tensile load only. No out-of plane loading, bending or torsion is applied. The aspect ratio of all laminates analyzed is $L/W = 10$ with $t/W < 1/20$, where L, W and t represent the length, width and thickness of the laminate, respectively. The laminae are assumed to be perfectly bounded and no slipping occurs within the laminate. Since the stress field is uniform through out the composite, only one quadrangular shell element with eight nodes is used to simulate the laminate in the numerical analysis.

In ABAQUS program, the local stresses and strains of the shell element in each lamina within the laminate can be automatically transformed to global coordinates. Basically, stresses and strains are calculated at each incremental step, and evaluated by the failure criteria to determine the occurrence of failure and the mode of failure. Mechanical



Boundary Conditions:
u = v = w = 0   at point A
v = w = 0   at point C
w = 0   on sides AB, BD, CD and AC

Figure 2. *Geometry and boundary conditions of the composite laminates.*

properties in the damaged area are reduced appropriately, according to the property degradation models. Stresses and strains will then be recalculated to determine any additional damage as a result of stress redistribution at the same load. This procedure will continue until no additional damage is found, and the next increment is then pursued. The final collapse load is determined when the composite plates cannot sustain any additional load.

In order to verify the proposed nonlinear failure analysis model, numerical results generated from the model are compared with the test data [2]. The material properties and strengths of Boron/Epoxy composites used in the calculation are:

Material properties:

$$E_{11e} = 207\,\text{GPa}, \quad E_{11p} = 180\,\text{GPa}, \quad E_{22e} = 21.2\,\text{GPa}, \quad E_{22p} = 15.9\,\text{GPa},$$
$$E_{22f} = -33.116\,\text{GPa}, \quad G_{12} = 7.25\,\text{GPa}, \quad \nu_{12} = 0.3$$

$$S_{6666} = \begin{cases} 15.198\,\text{GPa}^{-3} & \text{if constant} \\ 20.61 - 20\exp(-\gamma_{12}/0.00337)\ \text{GPa}^{-3} & \text{if variable.} \end{cases}$$

Yield Strengths:

$$X_{yt} = 828\,\text{MPa}, \quad X_{yc} = -1346\,\text{MPa}, \quad Y_{yt} = 57.9\,\text{MPa}, \quad Y_{yc} = -97.3\,\text{MPa}.$$

Ultimate strengths:

$$X_{ut} = 1370\,\text{MPa}, \quad X_{uc} = -2787\,\text{MPa}, \quad Y_{ut} = 86.3\,\text{MPa}, \quad Y_{uc} = -262\,\text{MPa}, \quad S = 128.6\,\text{MPa}.$$

It should be noted that the shear parameter S_{6666} has two types, a constant type and a variable type. The variable shear parameter is obtained by curve fitting from the pure shear test data [2], as shown in Figure 3. In this study, we use these two types of shear parameters in pure shear simulation to verify the proposed variable shear parameter can predict the shear behavior of laminates more accurately.

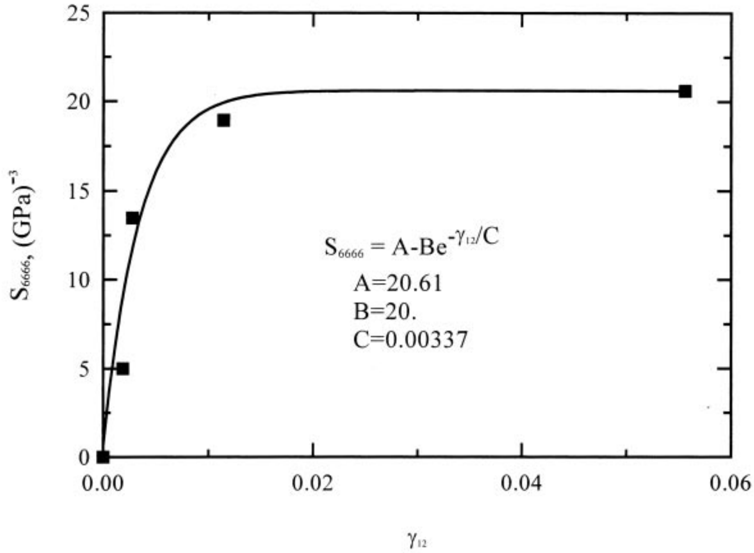


Figure 3. *The nonlinear shear parameter S_{6666} in various shear strain γ_{12} for the single Boron/Epoxy lamina.*

Verification of the Proposed Nonlinear Constitutive Model

It is necessary to assure that the proposed constitutive model can correctly simulate the stress–strain relations, in the principal directions and in pure shear of a lamina before predicting the mechanical behavior and failure stresses of composite laminates under various loading. Vaziri et al. [5] proposed an elastic–plastic constitutive model, which assumed that the material response could be adequately represented by bilinear, stress–strain curves in the principal directions and in pure shear for fiber-reinforced composite laminates. The main difference between the Vaziri's model and the proposed constitutive model is that the stress–strain relation in pure shear is assumed to be bilinear in the Vaziri's model and to be nonlinear (with a variable shear parameter) in the proposed model. Figure 4 shows the numerical results for a single lamina subjected to pure shear loading against the experimental data [2]. It is obvious that the shear stress–strain curve simulated by the proposed constitutive model agrees with the test data much better than that simulated by the Vaziri's model. The simulations of the stress–strain relations in principal directions by the Vaziri's model and the proposed model all agree very well with the test data [22] and are not shown here due to limited space.

The results simulated by the variable S_{6666} model and the constant S_{6666} model for a $[+45/-45]_s$ laminate subjected to uniaxial tension loading are shown in Figure 5. It can be seen that the result simulated by the proposed variable S_{6666} model exhibits better fit with the test data than that simulated by the constant S_{6666} model. Therefore, the proposed nonlinear constitutive model with elastic–plastic behavior in the principal directions and with nonlinear behavior, described by variable shear parameter, in pure shear is verified to be a suitable and reliable model for simulating the stress–strain behavior of composite lamina under loading.

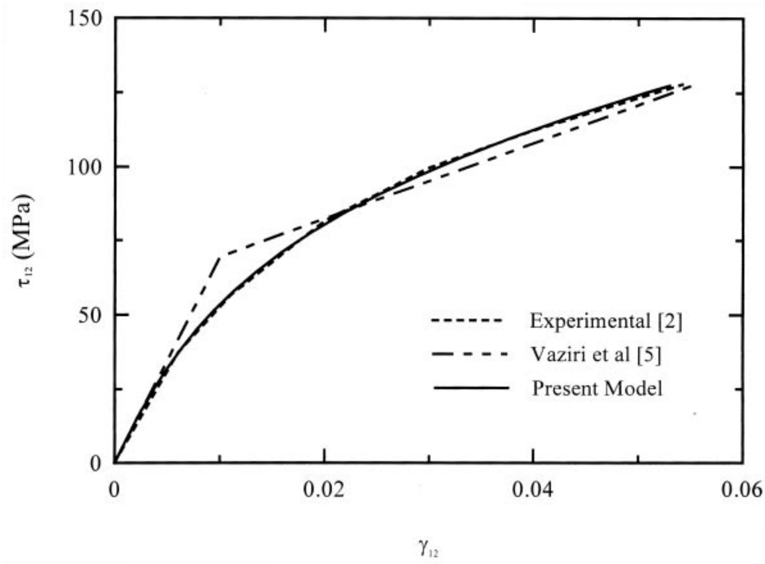


Figure 4. *Pure shear stress–strain curve for the single Boron/Epoxy lamina.*

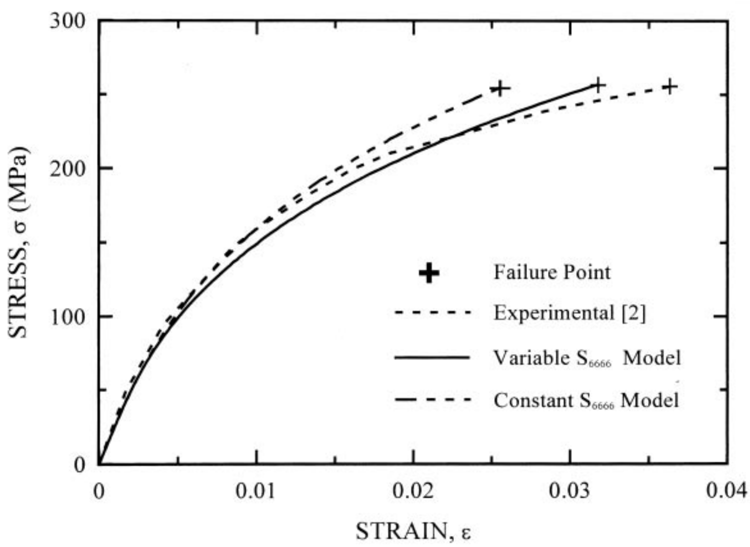


Figure 5. *Uniaxial tensile stress–strain curve simulated by various S_{6666} models for $[+45/-45]_S$ Boron/Epoxy laminate.*

Comparisons Among Various Post Failure Modes

The load–deformation behavior of a composite laminate is greatly affected by the stress–strain behavior of individual layer within the laminate and the ultimate strength of a composite laminate is greatly controlled by the post-damage mode of damaged lamina within the laminate. In order to verify the proposed post-damage mode in transverse direction of lamina is a suitable one, three idealized post failure modes, brittle, ductile and degrading modes, are taken into account. Figures 6–9 illustrate the distinct results simulated by these three post-damage modes for various $[+\theta/(\theta-90)]_s$ symmetrical composite laminates subjected to uniaxial tensile loading. For the $[0/90]_s$ laminate, as indicated in Figure 6, all the simulations by various post-damage modes seem to agree with the test data [2] very well before the initial damage point. However, after the initial damage point the stress–strain curve simulated by degrading mode shows better fit with the test data than those simulated by the other two modes. In the post-damage stage, the

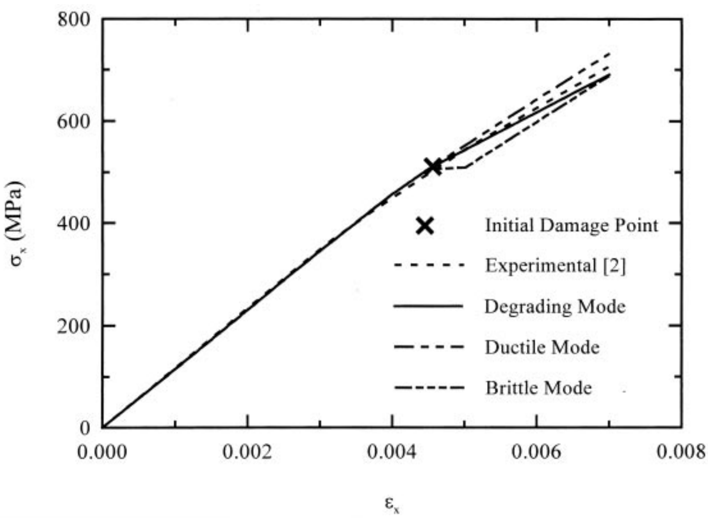


Figure 6. Uniaxial tensile stress–strain curve simulated by various matrix post failure modes for $[0/90]_S$ laminate.

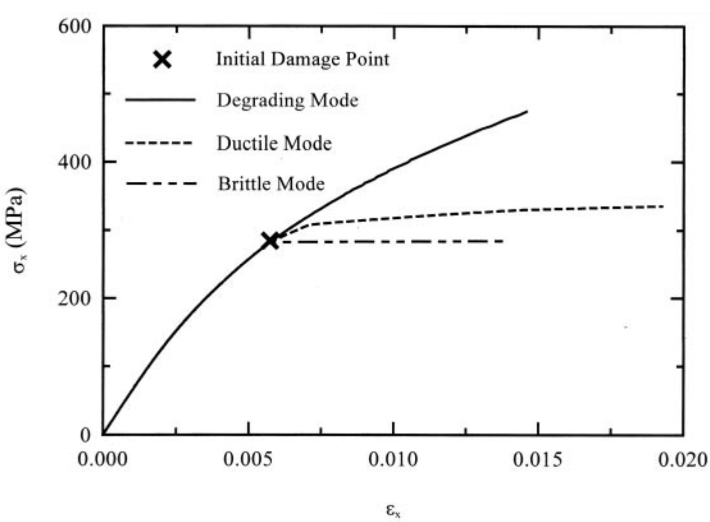


Figure 7. Uniaxial tensile stress–strain curve simulated by various matrix post failure models for $[+15/-75]_S$ laminate.

strength of the laminates is overestimated by the ductile mode and underestimated by the brittle mode. For the $[+15/-75]_s$ and $[+30/-60]_s$ laminates, as shown in Figures 7 and 8, although there are no test data to be compared with, they generally exhibit the same trend in numerical simulations, i.e., the ultimate stresses predicted by brittle and ductile modes are much lower than that predicted by the degrading mode. For $[+45/-45]_s$ composite laminate, shown in Figure 9, the result predicted by degrading mode shows better fit with the test data than those predicted by the other two modes. Therefore, the degrading mode

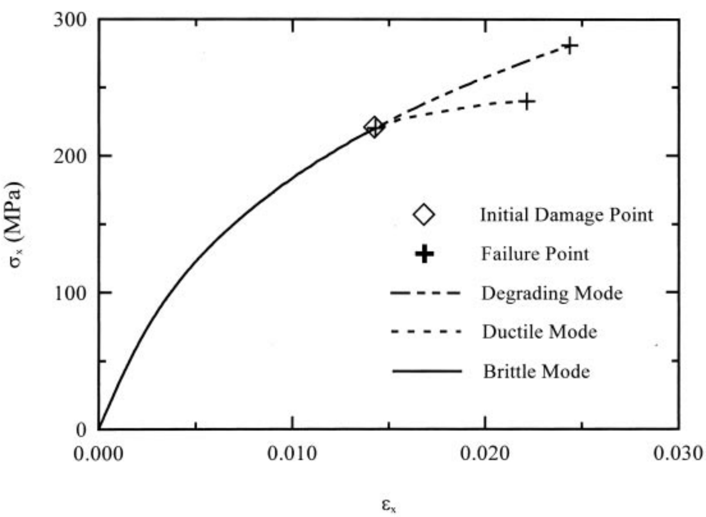


***Figure 8.** Uniaxial tensile stress–strain curve simulated by various matrix post failure modes for [+30/−60]S laminate.*

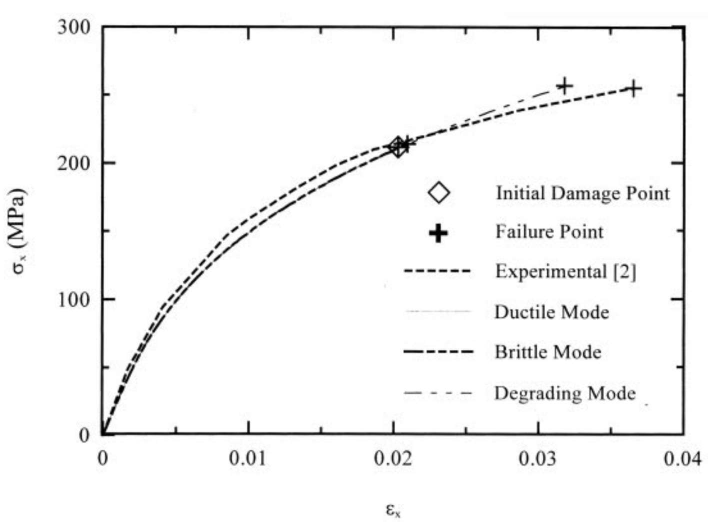


***Figure 9.** Uniaxial tensile stress–strain curve simulated by various matrix post failure modes for [+45/−45]S laminate.*

in tansverse direction of lamina is a more suitable post-damage mode than the other two modes and is used in the following numerical analyses. From Figures 7–9, we can also observe that the ultimate stress of the laminate predicted by ductile mode is close to that predicted by brittle mode as the angle θ of $[+\theta/(\theta-90)]_s$ laminate closes to 45°. In addition, the ultimate stresses of the laminates predicted by these two modes are always lower than that predicted by the degrading mode.

Comparisons Among Various Failure Criteria

As mentioned early, several failure criteria are widely used for composite materials, but there is no mechanic explanation why these criteria should work or what their limitations are. The Tsai–Wu failure criterion is one of the major quadratic failure criteria and has been extensively used in literature. To investigate the limitations of the Tsai–Wu failure criterion, the results predicted by the Tsai–Wu and the proposed mixed failure criteria are compared with each other.

Figures 10 and 11 illustrate the normalized failure stresses and failure contributions of various $[+\theta/-\theta]_s$ laminates under uniaxial tensile load with the Tsai–Wu failure criterion and the proposed mixed failure criterion. The value of the normalized failure stress with the Tsai–Wu failure criterion in 1-direction of the lamina is bigger than 1 for the cases of $0° < \theta < 20°$ (Figure 10). It means that the failure stress of fiber excesses its material strength and is not a reasonable phenomenon. The Tsai–Wu failure criterion assesses the failure of material by considering the failure contributions of all the material stresses, i.e., fiber stress, matrix stress and in-plane shear stress. When the $[+\theta/-\theta]_s$ laminates under uniaxial tensile load are in the cases of $0° < \theta < 20°$, the failure contributions of transverse stress (matrix stress) have negative values and the failure contributions of shear stress have lower positive values. Therefore, based on the Tsai–Wu failure criterion, the failure of the

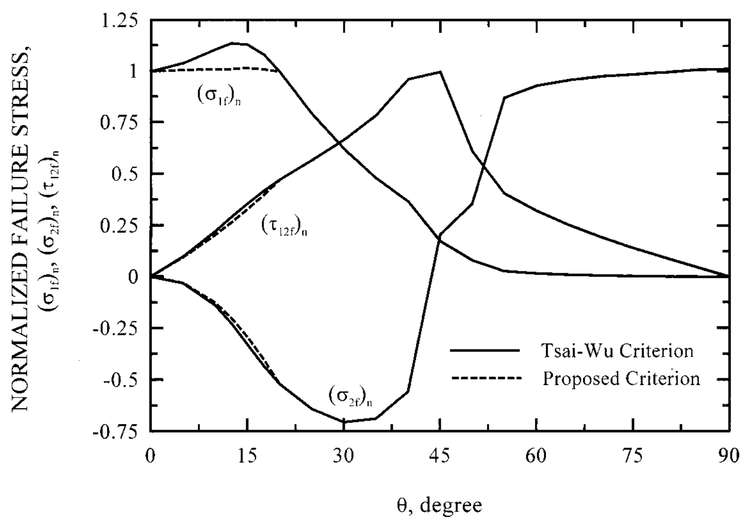


Figure 10. *The normalized material failure stresses predicted by Tsai–Wu and proposed failure criteria for various $[+\theta/-\theta]_s$ laminates.*

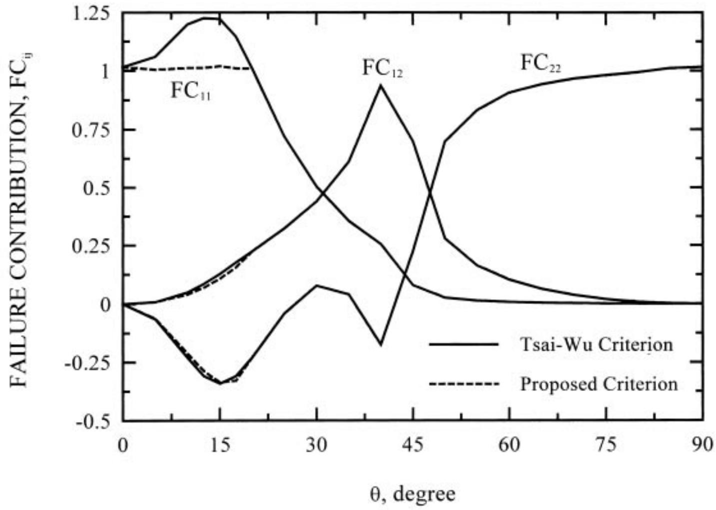


Figure 11. *The failure contributions of material stresses predicted by Tsai–Wu and proposed failure criteria for various $[+\theta/-\theta]_s$ laminates.*

laminate must occur at a high value of failure contribution in fiber stress to make the sum of various failure contributions equal to unity. In these cases, the failure contribution of fiber stress usually excesses the value 1 and it results in an overestimated failure stresses in fiber direction. To eliminate this unreasonable estimation, an extra limitation should be added into the Tsai–Wu failure criterion to obtain a more accurate and reasonable prediction. The proposed limitation in this study is $\sigma_1/X_{ut} \leq 1$ (as $\sigma_1 > 0$) or $\sigma_1/X_{uc} \leq 1$ (as $\sigma_1 < 0$). The combination of Tsai–Wu failure criterion with the proposed limitation is called the mixed failure criterion. The results predicted by mixed failure criterion in Figures 10 and 11 shows the reasonable values. In addition, Figure 11 indicates that the failure of laminate is induced by the fiber stress, matrix stress, in-plane shear stress, or any combination of these stresses for various $[+\theta/-\theta]_s$ laminates under uniaxial tensile load. When $0^\circ \leq \theta \leq 25^\circ$, the failure of laminate is mainly induced by fiber stress. For $25^\circ < \theta < 35^\circ$, the failure of laminate is induced by the combination of fiber stress and in-plane shear stress. In the cases of $35^\circ \leq \theta \leq 45^\circ$, the failure of laminate is primarily induced by in-plane shear stress. When $45^\circ < \theta < 50^\circ$, the failure of laminate is induced by the combination of in-plane shear stress and matrix stress. For $50^\circ \leq \theta \leq 90^\circ$, the failure of laminate is almost induced by matrix stress. It should be noted that the fiber stress and in-plane shear stress have the same normalized stress and failure contribution as failure occurring about at $\theta = 30^\circ$.

Figure 12 illustrates the comparison of uniaxial tensile stress–strain curves between various failure criteria with experimental data [2] for $[+30/-30]_s$ composite laminate. The results predicted by the Tsai–Wu and the proposed mixed failure criteria have the best agreement with the experimental data, and the result predicted by the Chang failure criterion has a better agreement with the experimental data than the maximum stress criterion does. As discussed preciously, the failure of $[+\theta/-\theta]_s$ laminate is mainly induced by the combination of fiber stress and in-plane shear stress when $25^\circ < \theta < 35^\circ$. Thus, for the $[+30/-30]_s$ laminate, the fiber stress and in-plane shear stress play the same important roles in determining the on set of failure. The maximum stress theory does not consider

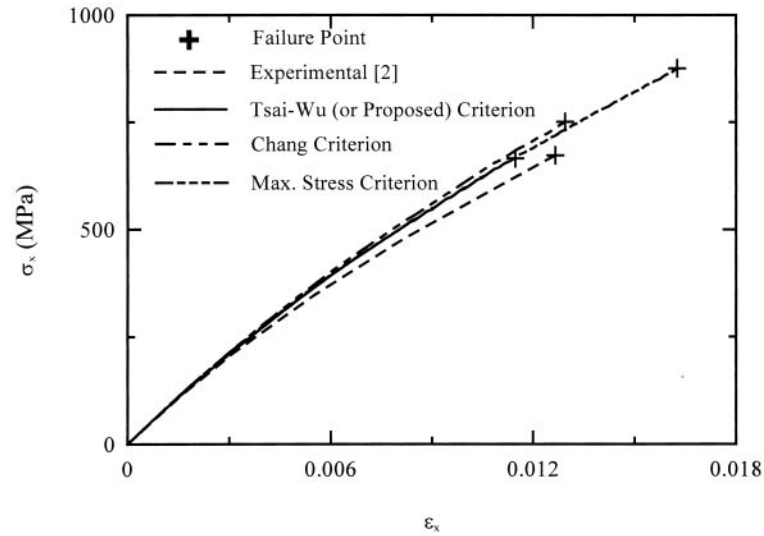


Figure 12. *Uniaxial tensile stress–strain curve simulated by various failure criteria for [+30/−30]$_S$ laminate.*

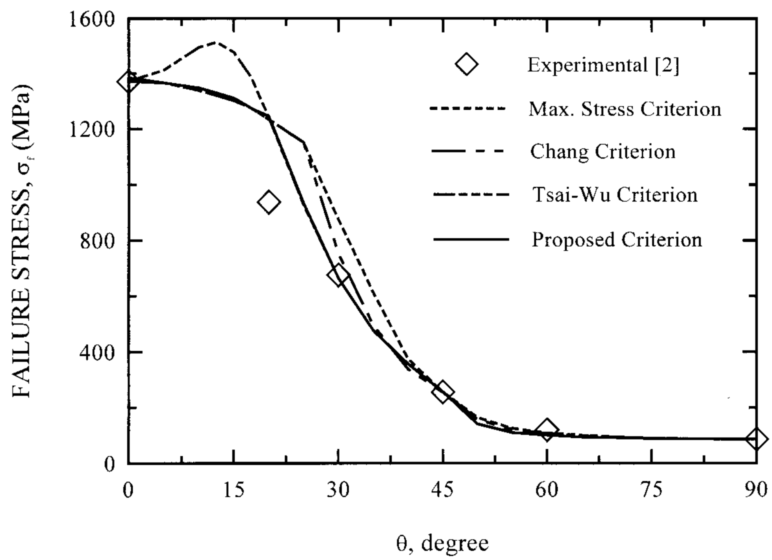


Figure 13. *Uniaxial tensile failure stress predicted by various failure criteria for various [+θ/−θ]$_S$ laminates.*

the coupling failure contribution of fiber stress and in-plane shear stress in the failure process. Hence, the failure stress of laminate in this case will be greatly overestimated due to only considering the failure of fiber stress or in-plane shear stress.

Figure 13 indicates the comparison of failure stresses for various $[+\theta/-\theta]_s$ laminates subjected to uniaxial tensile load with various failure criteria against experimental data [2]. In the cases of $0° \leq \theta \leq 25°$, the failure stresses predicted by maximum stress criterion, Chang failure criterion and the proposed mixed failure criterion are similar. But the failure stresses predicted by Tsai–Wu failure criterion are overestimated, especially at $\theta = 15°$. This is because that higher failure contributions of fiber stresses are needed to balance the negative failure contributions induced by matrix stresses. In the cases of $25° < \theta < 35°$, the failure stresses predicted by Tsai–Wu and proposed mixed failure criteria are similar because of using the same post-damage mode. The results predicted by maximum stress criterion and Chang failure criterion are overestimated. This is because that the maximum stress criterion does not consider the failure contributions coupled between fiber stress and in-plane shear stress. For Chang failure criterion, it is due to the use of the constant stiffness instead of degrading stiffness for matrix in the post-damage process. In the cases of $35° < \theta < 90°$, the results predicted by various failure criteria are very similar.

Figure 14 indicates the comparison of failure stresses for various $[(\theta)_3/(\theta+45)/(\theta-45)]_s$ laminates subjected to uniaxial tensile load with various failure criteria against experimental data [2]. The failure stresses predicted by Chang and the proposed mixed failure criteria are almost similar in all cases of θ and the results are in good agreement with the test data. In the cases of $0° \leq \theta \leq 15°$, the failure stresses are overestimated by Tsai–Wu failure criterion, especially at $\theta = 0°$. This is due to the interaction among $0°$ and $\pm 45°$ plies, causing the matrix stress in $0°$ ply to increase. As a result, large negative failure contributions of matrix stresses are induced and excessive positive failure contributions of fiber stresses are needed to balance them. The results predicted by the maximum stress criterion failure criterion exhibit two distinct trends in the cases of $10° < \theta < 60°$. The predicted failure stresses are usually underestimated when $10° < \theta < 38°$, but overestimated when $38° < \theta < 60°$. From Figures 13 and 14, it may conclude that the mixed failure criterion has not only reasonable predicting results but also better agreement with the test data.

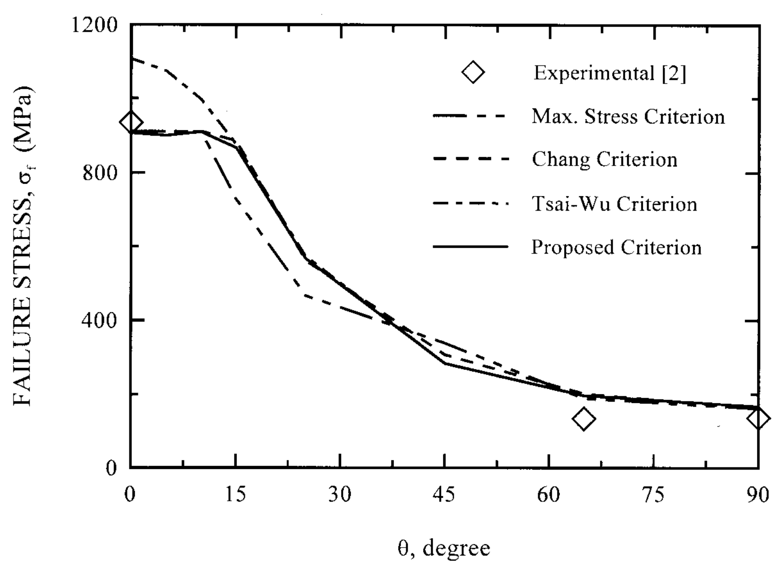


Figure 14. *Uniaxial tensile failure stress predicted by various failure criteria for various $[(\theta)_3/(\theta+45)/(\theta-45)]_s$ laminates.*

Comparisons Between Elastic–Plastic and Proposed Nonlinear Constitutive Models

Figures 15 and 16 illustrate the comparison of uniaxial tensile stress–strain curve for $[(0)_3/+45/-45]_s$ and $[(65)_3/+20/-70]_s$ laminates with Vaziri's elastic–plastic constitutive model [5] and the present model against the test data [2]. The mechanical responses of

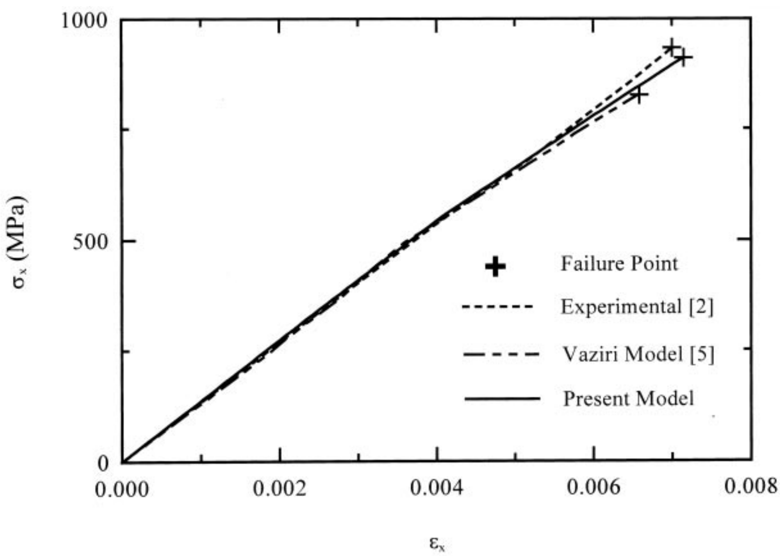


Figure 15. *Uniaxial tensile stress–strain curve simulated by Vaziri model and present model for* $[(0)_3/+45/-45]_s$ *laminate.*

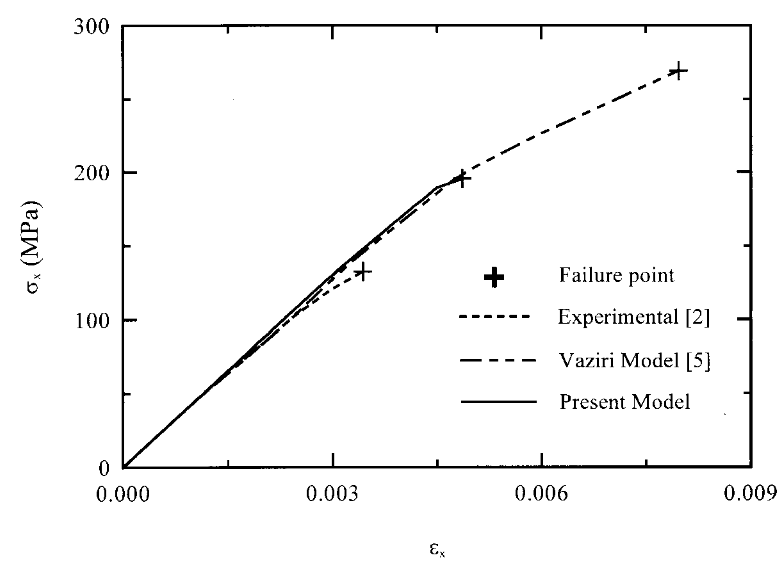


Figure 16. *Uniaxial tensile stress–strain curve simulated by Vaziri model and present model for* $[(65)_3/+20/-70]_s$ *laminate.*

these two types of laminates exhibit the interaction of normal stress and in-plane shear stress with neighboring laminae, especially for the $[(65)_3/+20/-70]_s$ laminate. The results simulated by the present model have better agreements with the test data than the Vaziri's model does. The results simulated by the Vaziri's model have fair agreements with the test data of $[(0)_3/+45/-45]_s$, laminate. However, due to the poor simulation on in-plane shear stress of the lamina, the results simulated by the Vaziri's model give very poor agreements with the test data of $[(65)_3/+20/-70]_s$ laminate. Thus, the present model is really a reasonable and accurate analysis model in predicting the stress–strain behavior as well as the ultimate stress of Boron/Epoxy composite laminates for various symmetrical staking sequences under uniaxial tensile load.

CONCLUSIONS

This paper presents a material constitutive model for simulating the mechanical response and predicting the ultimate strength of various symmetrical composite laminates subjected to uniaxial tensile load. The model is composed of three parts: (1) nonlinear constitutive model, (2) mixed failure criterion, and (3) post-damage mode. In the nonlinear constitutive model, the fiber and matrix are simulated by elastic–plastic behavior and the in-plane shear is simulated by nonlinear behavior with variable shear parameter, which is a function of in-plane shear strain. The mixed failure criterion is composed of the Tsai–Wu failure criterion and the maximum stress criterion to determine the damage onset of a lamina. The present failure criterion can eliminate the over estimation in fiber stresses as the negative failure contributions of matrix stresses occurring in the lamina. In the post-damage process, the fiber and the in-plane shear are simulated by a brittle mode and the matrix by a degrading mode.

The ultimate strengths predicted by the various failure criteria together with various post-damage modes against the experimental data show that the proposed mixed failure criterion and post-damage modes could predict the failure response of composite laminates under uniaxial tension more accurately than others. The numerical results simulated by Vaziri's elastic–plastic constitutive model and the present constitutive model against the test data illustrate that the present constitutive model can predict the stress–strain relation, especially for the interaction of normal stress and in-plane shear stress in neighboring laminae, more accurately. The favorable agreement between the present numerical predictions and experimental data demonstrates that the nonlinear constitutive model together with the mixed failure criterion and degrading post-damage modes is a useful tool in the nonlinear failure analysis of fiber-reinforced composite laminated materials. It shows the potential of the present nonlinear failure approach.

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