Influence of Geometry and End Conditions on Optimal Fundamental Natural Frequencies of Symmetrically Laminated Plates

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ABSTRACT: The fundamental natural frequency of rectangular symmetrically laminated plates with a given material system is maximized with respect to fiber orientations by using a sequential linear programming method together with a simple move-limit strategy. Significant influence of plate thickness, aspect ratio, circular cutout and end conditions on the optimal fiber orientations and the associated optimal fundamental natural frequency of fiber-reinforced laminated composite plates has been shown through this investigation.

1. INTRODUCTION

THE APPLICATIONS OF fiber-composite laminate materials to primary components in advanced structures such as spacecraft, high-speed aircraft, and satellite, have increased rapidly in recent years. Since most major components of the aerospace structures are made of plates, a knowledge of dynamic characteristics of fiber-reinforced laminated composite plates, such as their fundamental natural frequency, is essential [1–3].

The fundamental natural frequency of fiber-reinforced laminated composite plate highly depends on end conditions [4], lamination parameters such as ply orientations [5–7], and geometric variables such as aspect ratio, thickness and cutout [8–11]. Therefore, for composite plates with a given material system, geometric shape, thickness and end condition, the proper selection of appropriate lamination to maximize the fundamental natural frequency of the plates becomes a crucial problem. However, in spite of the high potential for improved dynamic performance by use of composite optimization, there has not been very much activity in this area [12].

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Research on the subject of structural optimization has been reported by many investigators [13]. Among various optimization schemes, the method of sequential linear programming has been successfully applied to many large scale structural problems [14–15]. Hence, linearization of nonlinear optimization problems to meet requirements for iterative applications of a linear programming method is one of the most popular approaches to solve the structural optimization problem.

In this investigation, optimization of symmetrically laminated plates for their fundamental natural frequencies with respect to fiber orientations is performed by using a sequential linear programming method together with a simple move-limit strategy. The fundamental natural frequencies of composite plates are calculated by using the ABAQUS finite element program [16]. In the paper, the constitutive equations for fiber-composite laminate, vibration analysis and optimization method are briefly reviewed. Then, the influence of end conditions, plate thicknesses, aspect ratios, and cutouts on the optimal fiber orientations and the optimal fundamental natural frequencies of composite plates is presented. Finally, important conclusions obtained from the study are given.

2. CONSTITUTIVE MATRIX FOR FIBER-COMPOSITE LAMINAE

In the finite element analysis, the laminate plates are modeled by eight-node isoparametric laminate shell elements with six degrees of freedom per node (three displacements and three rotations). The formulation of the shell allows transverse shear deformation [16–17]. These shear flexible shells can be used for both thick and thin shell analyses [16].

During the analysis, the constitutive matrices of composite materials at element integration points must be calculated before the stiffness matrices are assembled from element level to global level. For fiber-composite laminate materials, each lamina can be considered as an orthotropic layer in a plane stress condition (Figure 1). The stress-strain relations for a lamina in the material coordinates (1,2,3) at an integration point can be written as

$$\{\sigma'\} = [Q_1'_{}]\{\epsilon'\} \tag{1}$$

$$\{\tau_t'\} = [Q_2']\{\gamma_t'\} \tag{2}$$

$$[Q_1'] = \begin{bmatrix} \dfrac{E_{11}}{1 - \nu_{12}\nu_{21}} & \dfrac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} & 0 \\ \dfrac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}} & \dfrac{E_{22}}{1 - \nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \tag{3}$$

$$[Q_2'] = \begin{bmatrix} \alpha_1 G_{13} & 0 \\ 0 & \alpha_2 G_{23} \end{bmatrix} \tag{4}$$

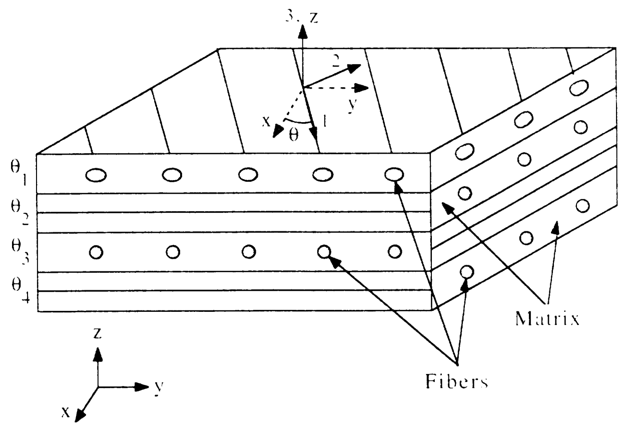


Figure 1. ***Material and element coordinate systems for fiber-composite laminate.***

where $\{\sigma'\} = \{\sigma_1,\sigma_2,\tau_{12}\}^T,\{t_t'\} = \{\tau_{13},\tau_{23}\}^T$, $\{\epsilon'\} = \{\epsilon_1,\epsilon_2,\gamma_{12}\}^T,\{\gamma_t'\} = \{\gamma_{13}, \gamma_{23}\}^T$. The α_1 and α_2 are shear correction factors. In ABAQUS, the shear correction factors are calculated by assuming that the transverse shear energy through the thickness of laminate is equal to that of unidirectional bending [16,18].

The constitutive equations for the lamina in the element coordinates (x,y,z) then become

$$\{\sigma\} = [Q_1]\{\epsilon\}, \qquad [Q_1] = [T_1]^T[Q_1'][T_1] \tag{5}$$

$$\{\tau_t\} = [Q_2]\{\gamma_t\}, \qquad [Q_2] = [T_2]^T[Q_2'][T_2] \tag{6}$$

$$[T_1] = \begin{bmatrix} \cos^2\phi & \sin^2\phi & \sin\phi\cos\phi \\ \sin^2\phi & \cos^2\phi & -\sin\phi\cos\phi \\ -2\sin\phi\cos\phi & 2\sin\phi\cos\phi & \cos^2\phi - \sin^2\phi \end{bmatrix} \tag{7}$$

$$[T_2] = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \tag{8}$$

where $\{\sigma\} = \{\sigma_x,\sigma_y,\tau_{xy}\}^T,\{\tau_t\} = \{\tau_{xz},\tau_{yz}\}^T,\{\epsilon\} = \{\epsilon_x,\epsilon_y,\gamma_{xy}\}^T,\{\gamma_t\} = \{\gamma_{xz},\gamma_{yz}\}^T$, and ϕ is measured counterclockwise from the element local x-axis to the material 1-axis. Assume $\{\epsilon_0\} = \{\epsilon_{x0},\epsilon_{y0},\gamma_{xy0}\}^T$ are the in-plane strains at the mid-surface of the laminate section and $\{\varkappa\} = \{\varkappa_x,\varkappa_y,\varkappa_{xy}\}^T$ are the curvatures. The in-plane strains at a distance, z, from the mid-surface become

$$\{\epsilon\} = \{\epsilon_0\} + z\{\varkappa\} \tag{9}$$

If h is the total thickness of the section, the stress resultants, $\{N\} = \{N_x, N_y,$

$N_{xy}\}^T$, $\{M\} = \{M_x, M_y, M_{xy}\}^T$ and $\{V\} = \{V_x, V_y\}^T$, can be defined as

$$\{N\} = \int_{-h/2}^{h/2} \{\sigma\} dz = \int_{-h/2}^{h/2} [Q_1](\{\epsilon_0\} + z\{\varkappa\}) dz \tag{10}$$

$$\{M\} = \int_{-h/2}^{h/2} z\{\sigma\} dz = \int_{-h/2}^{h/2} z[Q_1](\{\epsilon_0\} + z\{\varkappa\}) dz \tag{11}$$

$$\{V\} = \int_{-h/2}^{h/2} \{\tau_t\} dz = \int_{-h/2}^{h/2} [Q_2]\{\gamma_t\} dz \tag{12}$$

If there are n layers in the layup, the above equations can be rewritten as a summation of integrals over the n laminae in the following forms:

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{V\} \end{Bmatrix} = \sum_{j=1}^{n} \begin{bmatrix} (z_{jt} - z_{jb})[Q_1] & \frac{1}{2}(z_{jt}^2 - z_{jb}^2)[Q_1] & [0] \\ \frac{1}{2}(z_{jt}^2 - z_{jb}^2)[Q_1] & \frac{1}{3}(z_{jt}^3 - z_{jb}^3)[Q_1] & [0] \\ [0]^T & [0]^T & (z_{jt} - z_{jb})[Q_2] \end{bmatrix} \begin{Bmatrix} \{\epsilon_0\} \\ \{\varkappa\} \\ \{\gamma_t\} \end{Bmatrix} \tag{13}$$

where z_{jt} and z_{jb} are the distance from the mid-surface of the section to the top and the bottom of the jth layer respectively. The [0] is a 3 by 2 matrix with all the coefficients equal to zero. It should be noted that for laminate section with symmetric layup, the extensional and the flexural terms in the constitutive matrix [Equation (13)] become uncoupled, i.e.,

$$\sum_{j=1}^{n} \frac{1}{2}(z_{jt}^2 - z_{jb}^2)[Q_1] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{14}$$

3. VIBRATION ANALYSIS

For the finite-element analysis of an undamped structure, if there are no external forces, the equation of motion of the structure can be written in the following form [19]:

$$[M]\{\ddot{D}\} + [K]\{D\} = \{0\} \tag{15}$$

where $\{D\}$ is a vector containing the unrestrained nodal degrees of freedoms, $[M]$ a structural mass matrix, $[K]$ a structural stiffness matrix, and $\{0\}$ a zero vector. Since $\{D\}$ undergoes harmonic motion, the vectors $\{D\}$ and $\{\ddot{D}\}$ become

$$\{D\} = \{\bar{D}\} \sin \omega t; \qquad \{\ddot{D}\} = -\omega^2\{\bar{D}\} \sin \omega t \tag{16}$$

where $\{\bar{D}\}$ vector contains the amplitudes of $\{D\}$ vector and ω is the frequency. Then Equation (15) can be written in an eigenvalue expression as

$$([K] - \lambda[M])\{\bar{D}\} = \{0\} \tag{17}$$

where $\lambda = \omega^2$ is the eigenvalue and $\{\bar{D}\}$ becomes the eigenvector. In ABAQUS, a subspace iteration procedure [20] is used to solve for the eigenvalues, the natural frequency, and the eigenvectors. The obtained smallest natural frequency (fundamental natural frequency) is then the objective function for maximization.

4. SEQUENTIAL LINEAR PROGRAMMING

A general optimization problem may be defined as the following:

Maximize:

$$f(\underline{x}) \tag{18a}$$

Subjected to:

$$g_i(\underline{x}) \leq 0, \qquad i = 1, \ldots, r \tag{18b}$$

$$h_j(\underline{x}) = 0, \qquad j = r + 1, \ldots, m \tag{18c}$$

$$p_k \leq x_k \leq q_k, \qquad k = 1, \ldots, n \tag{18d}$$

where $f(\underline{x})$ is an objective function, $g_i(\underline{x})$ are inequality constraints, $h_j(\underline{x})$ are equality constraints, and $\underline{x} = \{x_1, x_2, \ldots, x_n\}^T$ is a vector of design variables.

For the general optimization problem of Equations (18a)–(18d), a linearized problem may be constructed by approximating the nonlinear functions about a current solution point, $\underline{x}_0 = \{x_{01}, x_{02}, \ldots, x_{0n}\}^T$, in a first-order Taylor series expansion as follows

Maximize

$$f(\underline{x}) = f(\underline{x}_0) + \nabla f(\underline{x}_0)^T \delta \underline{x} \tag{19a}$$

Subjected to

$$g_i(\underline{x}) = g_i(\underline{x}_0) + \nabla g_i(\underline{x}_0)^T \delta \underline{x} \leq 0 \tag{19b}$$

$$h_j(\underline{x}) = h_j(\underline{x}_0) + \nabla h_j(\underline{x}_0)^T \delta \underline{x} = 0 \tag{19c}$$

$$p_k \leq x_k \leq q_k \tag{19d}$$

where $i = 1, \ldots, r$; $j = r + 1, \ldots, m$; $k = 1, \ldots, n$; $\delta \underline{x} = \{x_1 - x_{01}, x_2 - x_{02}, \ldots, x_n - x_{0n}\}^T$.

It is clear that Equations (19a)–(19d) represent a linear programming problem where variables are contained in the vector $\delta \underline{x}$. A solution for Equations (19a)–(19d) may be easily obtained by the simplex method [21]. After obtaining a solution of Equations (19a)–(19d), say $\underline{x}_1$, we can linearize the original problem, Equations (18a)–(18d), at $\underline{x}_1$ and solve the new linear programming problem. The process is repeated until a precise solution is achieved. This approach is referred to as sequential linear programming [14–15].

Although the procedure for a sequential linear programming is simple, difficulties may arise during the iterations. First, the optimum solution for the approximate linear problem may violate the constraint conditions of the original optimization problem. Second, in a nonlinear problem, the true optimum solution may appear between two constraint intersections. A straightforward successive linearization may lead to an oscillation of the solution between the widely separated values. Difficulties in dealing with such a problem may be avoided by imposing a "move limit" on the linear approximation [14–15]. The concept of a move limit is that a set of box-like admissible constraints are placed in the range of $\delta \underline{x}$ and it should gradually approach zero as the iterative process continues. It is known that computational economy and accuracy of the approximate solution may depend greatly on the choice of the move limit. In general, the choice of a suitable move limit depends on experience and also on the results of previous steps.

The algorithm of the sequential linear programming with selected move limits may be summarized as follows:

1. Linearize the nonlinear objective function and associated constraints with respect to an initial guess $\underline{x}_0$.
2. Impose move limits in the form of $-\underline{S} \le (\underline{x} - \underline{x}_0) \le \underline{R}$, where $\underline{S}$ and $\underline{R}$ are properly chosen lower and upper bounds.
3. Solve the approximate linear programming problem to obtain an optimum solution $\underline{x}_1$.
4. Repeat the procedures from (1) to (3) by redefining $\underline{x}_1$ with $\underline{x}_0$ until either the subsequent solutions do not change significantly (i.e., true convergence) or the move limit approaches zero (i.e., forced convergence). If the solution obtained is due to forced convergence, the procedures from (1) to (4) should be repeated with another initial guess.

5. NUMERICAL ANALYSIS

5.1 Rectangular Laminate Plates with Various Aspect Ratios and End Conditions

In this section composite laminate rectangular plates with three types of end conditions are considered, which are two edges simply supported (denoted by S) and two edges fixed (denoted by F), four edges simply supported, and four edges

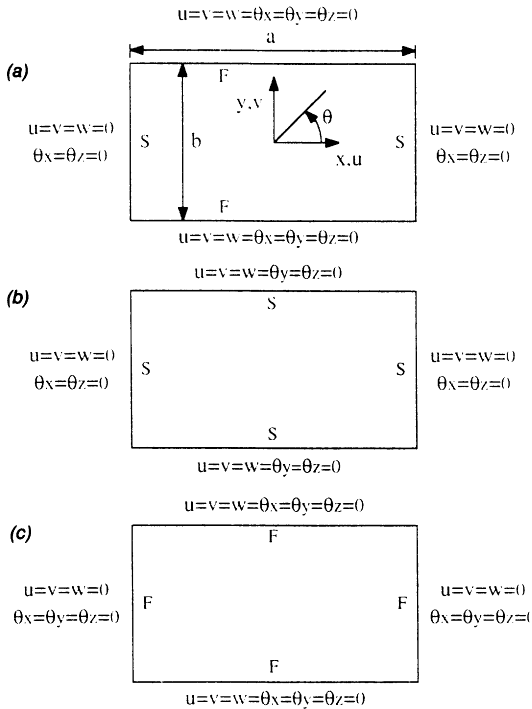


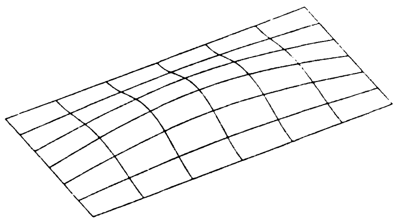
Figure 2. *Composite plates with (a) two simply supported ends and two fixed ends, (b) four simply supported ends, and (c) four fixed ends.*

fixed (Figure 2). The width of the plates, b, is 10 cm and the length of the plates, a, are varied between 5 cm and 30 cm. The thickness of each ply is 0.125 mm. The laminate layups of the plates are $[\pm\theta/90/0]_{ns}$. In order to study the influence of plate thickness on the results of optimization, $n = 2$ (16-plies thin plate) and 10 (80-plies thick plate) are selected for analysis. The lamina is consisted of Graphite/Epoxy (Hercules AS/3501-6) and material constitutive properties are taken from Crawley [6], which are $E_{11} = 128$ GPa, $E_{22} = 11$ GPa, $\nu_{12} = 0.25$, $G_{12} = G_{13} = 4.48$ GPa, $G_{23} = 1.53$ GPa, $\varrho = 1.5 \times 10^3$ kg/m^3. In the finite element analysis, no symmetry simplifications are made. A typical first vibration mode for $[\pm 40/90/0]_{2s}$ rectangular laminate plate with a/b = 2 and with two simply supported ends and two fixed ends is shown in Figure 3(a).

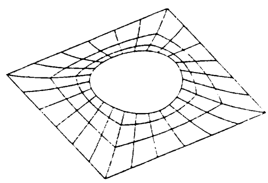
Based on the sequential linear programming method, in each iteration the current linearized optimization problem becomes:

Maximize

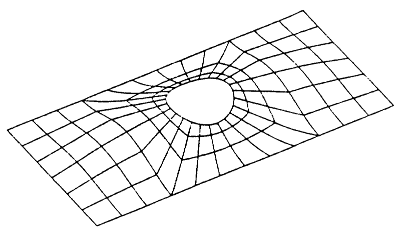
$$\omega(\theta) = \omega(\theta_0) + (\theta - \theta_0)\frac{\partial\omega}{\partial\theta}\bigg|_{\theta = \theta_0} \tag{20a}$$



(a) Rectangular plate (a/b = 2), ω = 7736 rad/sec



(b) Square plate (d/a = 0.5), ω = 13217 rad/sec



(c) Rectangular plate (a/b = 2, d/b = 0.4), ω = 8246 rad/sec

Figure 3. *Typical first vibration modes of $[\pm 40/90/0]_{2s}$ composite plates with two simply supported ends and two fixed ends.*

Subjected to

$$0^\circ \leq \theta \leq 90^\circ \tag{20b}$$

$$-r \times q \times 0.5^s \leq (\theta - \theta_o) \leq r \times q \times 0.5^s \tag{20c}$$

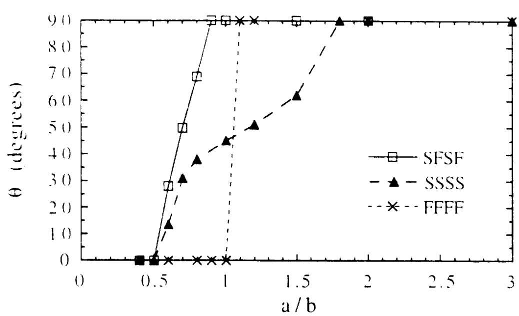
where ω is the fundamental natural frequency. The θ_0 is a solution obtained in the previous iteration. The r and q in Equation (20c) are the size and the reduction rate of the move limit. In the present study, the values of r and q are selected to be 20° and $0.9^{(N-1)}$, where N is a current iteration number. In order to control the oscillation of the solution, a parameter 0.5^s is introduced in the move limit, where s is the number of oscillation of the derivative $\partial\omega/\partial\theta$ that has taken place before the current iteration. The value of s increases by 1 if the sign of $\partial\omega/\partial\theta$ changes. Whenever oscillation of the solution occurs, the range of the move limit is reduced to half of its current value, which is similar to a bisection method [22]. This expedites the solution convergent rate very rapidly.

The $\partial\omega/\partial\theta$ term in Equation (20a) may be approximated by using a forward finite-difference method with the following form:

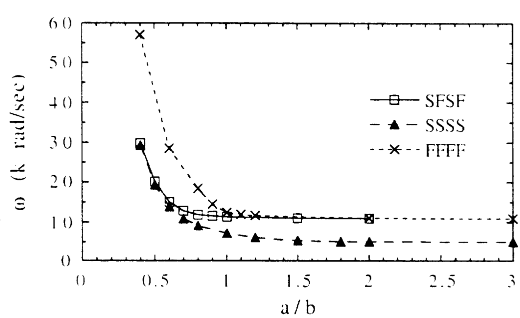
$$\frac{\partial\omega}{\partial\theta} = \frac{[\omega(\theta_0 + \Delta\theta) - \omega(\theta_0)]}{\Delta\theta} \tag{21}$$

Hence, in order to determine the value of $\partial\omega/\partial\theta$ numerically, two finite element analyses to compute $\omega(\theta_0)$ and $\omega(\theta_0 + \Delta\theta)$ are needed in each iteration. In this study, the value of $\Delta\theta$ is selected to be 1° in most iterations.

Figure 4 shows the optimal fiber angle θ and the associated optimal fundamental natural frequency ω with respect to the plate aspect ratio a/b for $[\pm\theta/90/0]_{2s}$ rectangular composite plates. From Figure 4(a), we can see that the results of optimization for these plates with different end conditions exhibit similar trends. With the increasing of the plate aspect ratio a/b, the optimal fiber angles all change from 0° to 90°. However, this transitional range in the aspect ratio is the widest for plates with four simply supported ends (say 0.5 ≤ a/b ≤ 1.8), and the narrowest for plates with four fixed ends (say a/b around 1). The results in Figure 4(b) show that the effect of end conditions on the optimal fundamental

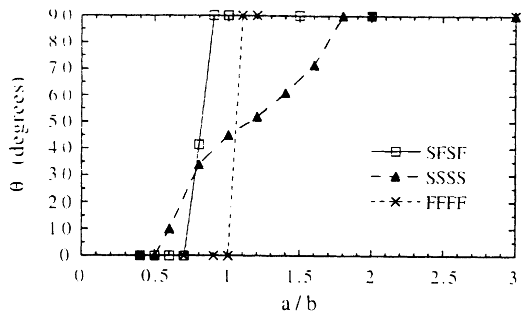


(a) Aspect ratio a/b vs. optimal fiber angle θ

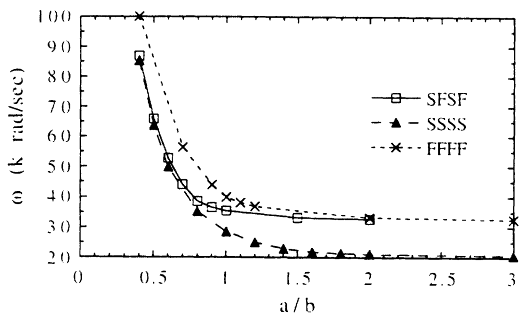


(b) Aspect ratio a/b vs. optimal fundamental natural frequency ω

Figure 4. Effect of end conditions and plate aspect ratios on optimal fiber angle and optimal fundamental natural frequency of $[\pm\theta/90/0]_{2s}$ rectangular composite plates.



(a) Aspect ratio a/b vs. optimal fiber angle θ



(b) Aspect ratio a/b vs. optimal fundamental natural frequency ω

Figure 5. ***Effect of end conditions and plate aspect ratios on optimal fiber angle and optimal fundamental natural frequency of*** $[\pm\theta/90/0]_{10s}$ ***rectangular composite plates.***

natural frequencies are more pronounced for values with a/b < 1. As a/b increases, the optimal fundamental natural frequencies attenuate to constant values. Overall, the highest optimal fundamental natural frequencies are exhibited by the plates with four fixed ends and the lowest by those with four simply support ends, which is consistent with the trend exhibited by corresponding plates with a fixed value of θ. When a/b ratio is small, the optimal fundamental natural frequencies of plates with two simply supported ends and two fixed ends are similar to those of plates with four simply supported ends. However, when a/b ratio is large, they are close to those of plates with four fixed ends.

Figure 5 shows the optimal fiber angle θ and the associated optimal fundamental natural frequency versus the plate aspect ratio a/b for $[\pm\theta/90/0]_{10s}$ rectangular composite plates. Comparing Figure 5(a) with Figure 4(a), we can observe that the optimal fiber orientations of the thick plates with four simply supported ends and with four fixed ends are almost the same as those of thin plates with corresponding end conditions. The plate thickness only has some influence on plates with two simply supported ends and two fixed ends. Figure 5(b) of thick plates shows the similar trend as Figure 4(b) of thin plates, except the values of the optimal fundamental natural frequencies for thick plates are much higher than those of thin plates.

5.2 Square Laminate Plates with Various Central Circular Cutouts and End Conditions

In this section, composite laminate square plates with a central circular cutout as shown in Figure 6(a) are analyzed. The length of the plates, a, is set to 10 cm and the diameter of the cutout, d, are varied between 1 cm and 8 cm. Three types of end conditions, i.e., SFSF, SSSS, and FFFF, similar to those in previous section are considered. Again, the laminate layups, $[\pm\theta/90/0]_{2s}$ and $[\pm\theta/90/0]_{10s}$, are selected for analysis. A typical first vibration mode for $[\pm 40/90/0]_{2s}$ square laminate plate with d/a = 0.5 and with two simply supported ends and two fixed ends is shown in Figure 3(b).

Figure 7 shows the optimal fiber angle θ and the associated optimal fundamental natural frequency versus the ratio d/a for $[\pm\theta/90/0]_{2s}$ square composite plates. From Figure 7(a), we can see that the optimal fiber orientations of the plates with four simply supported ends are insensitive to the sizes of cutouts and always remain 45°. Nevertheless, the optimal fiber orientations of the plates with four fixed ends and with two simply supported end and two fixed ends show significant sensitivity to the sizes of cutouts, and it seems that when the holes become large, these optimal fiber angles gradually approach to 45°. Figure 7(b) shows that the optimal fundamental natural frequencies increase with the increase of the sizes of cutouts for plates with different end conditions. This phenomenon is quite different from our intuition that introducing a large hole into a plate can cause a

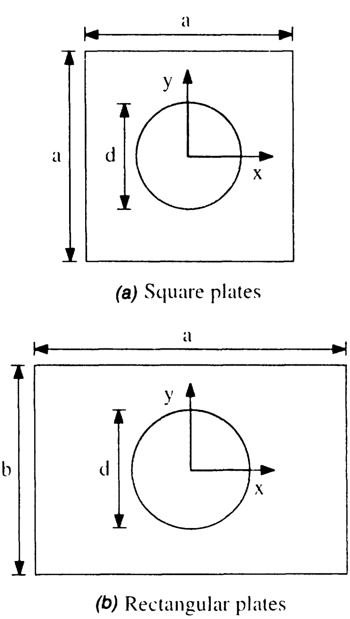
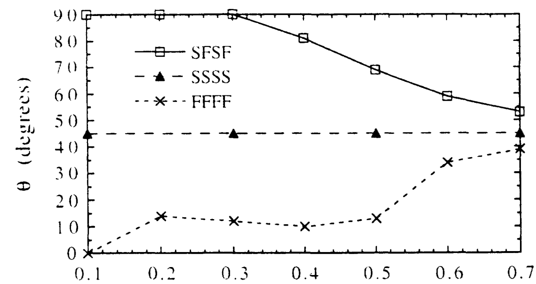
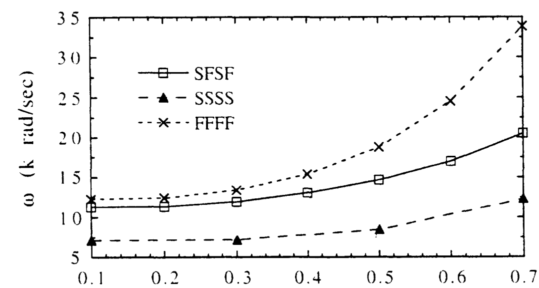


***Figure 6.** Composite plates with a circular cutout at center.*



(a) d/a vs. optimal fiber angle θ



(b) d/a vs. vs. optimal fundamental natural frequency ω

Figure 7. ***Effect of end conditions and cutouts on optimal fiber angle and optimal fundamental natural frequency of*** $[\pm\theta/90/0]_{2s}$ ***square composite plates with circular cutouts.***

reduction in the fundamental natural frequency of the plate. However, past research did show that introducing a hole into a composite plate does not always reduce the fundamental natural frequency and, in some instances, may increase its fundamental natural frequency [23,24]. This is because that the fundamental natural frequency of a composite plate is not only influenced by cutout, but also influenced by material orthotropy, end condition, and plate geometry.

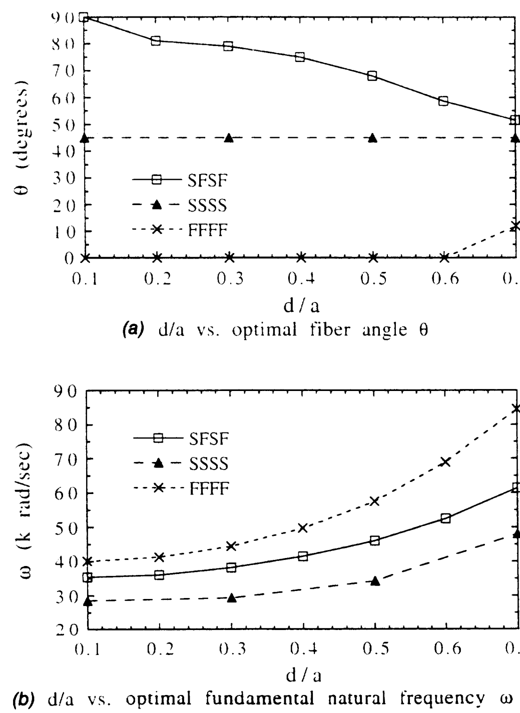
Figure 8 shows the optimal fiber angle θ and the associated optimal fundamental natural frequency with respect to the d/a ratio for $[\pm\theta/90/0]_{10s}$ square composite plates. From Figure 8(a), we can see that only the optimal fiber angles of plates with two simply supported ends and two fixed ends are sensitive to the sizes of cutouts. From Figure 8(b) we can again observe that the optimal fundamental natural frequencies of thick plates increase with the increase of the sizes of cutouts.

Comparing Figure 8 with Figure 7, we can observe that the plate thickness only affects the optimal fiber orientations of plates with four fixed ends. It has very little influence on the optimal fiber orientations of plates with four simply supported ends and with two simply supported ends and two fixed ends. In addition, the values of the optimal fundamental natural frequencies for thick plates are much higher than those of thin plates.

5.3 Rectangular Laminate Plates Containing a Central Circular Cutout with Various Aspect Ratios and End Conditions

In this section composite laminate rectangular plates with a central circular cutout as shown in Figure 6(c) are analyzed. The width of the plates, b, is 10 cm, the length of the plates, a, varies between 5 cm and 40 cm, and the diameter of the cutout, d, is selected to be 4 cm. Again, three types of end conditions, SFSF, SSSS, and FFFF, and two types of laminate layups, $[\pm\theta/90/0]_{2s}$ and $[\pm\theta/90/0]_{10s}$, are selected for analysis. A typical first vibration mode for $[\pm 40/90/0]_{2s}$ rectangular laminate plate with a/b = 2, d/b = 0.4, and with two simply supported ends and two fixed ends is shown in Figure 3(c).

Figures 9 and 10 shows the optimal fiber angle and the associated optimal fundamental frequency versus the plate aspect ratio a/b for $[\pm\theta/90/0]_{2s}$ and $[\pm\theta/90/0]_{10s}$ rectangular composite plates with a central circular cutout. From Figures 9(a) and 10(a) we can see that the results of optimization for these plates with different end conditions exhibit similar trends. When the plate aspect ratios are increased, the optimal fiber angles of these plates all change from minimum values to 90°. Similar to the rectangular plates without cutouts, the transitional



(a) d/a vs. optimal fiber angle θ

(b) d/a vs. optimal fundamental natural frequency ω

Figure 8. *Effect of end conditions and cutouts on optimal fiber angle and optimal fundamental natural frequency of $[\pm\theta/90/0]_{10s}$ square composite plates with circular cutouts.*

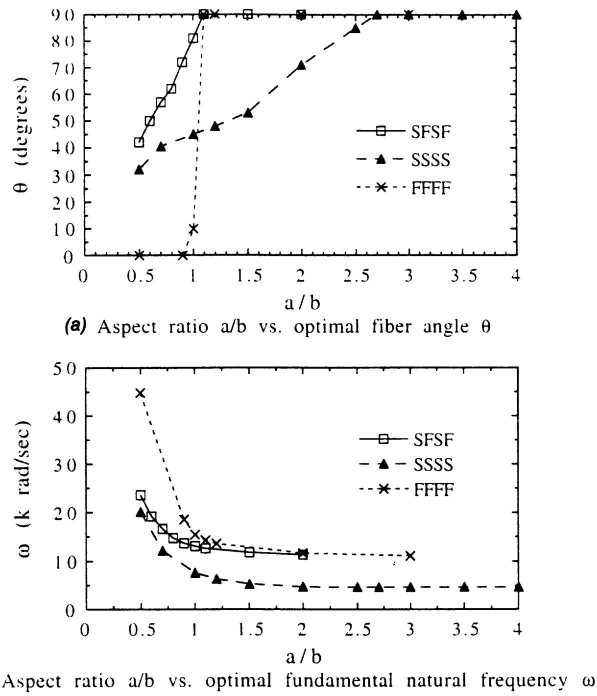


Figure 9. *Effect of end conditions and plate aspect ratios on optimal fiber angle and optimal fundamental natural frequency of $[\pm\theta/90/0]_{2s}$ rectangular composite plates with circular cutouts.*

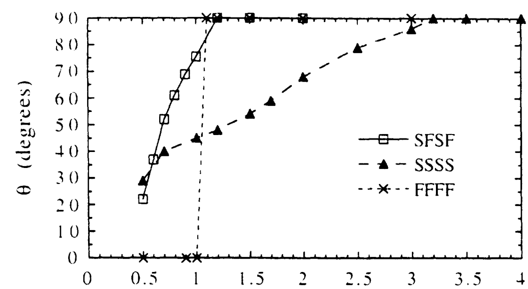
range in the aspect ratio is the widest for plates with four simply supported ends, and the narrowest for plates with four fixed ends. Comparing Figure 9(a) with Figure 10(a), we can observe that the plate thickness has almost no influence on the optimal fiber angles of plates with four fixed ends. The plate thickness only has influence on the optimal fiber angles of plates with two simply supported ends and two fixed ends for a/b below 1, and of plates with four simply supported ends for a/b around 3.

Comparing Figure 9(a) with Figure 4(a), we can see that the cutout has significant influence on the optimal fiber angles of thin plates with four simply supported ends and with two simply supported ends and two fixed ends. However, the cutout has no influence on the optimal fiber angles of thin plates with four fixed ends. Similar results can be observed for thick plates if we compare Figure 10(a) with Figure 5(a). By comparing Figure 9(b) with Figure 4(b), or Figure 10(b) with Figure 5(b), we can see that the introduction of cutout generally increases the optimal fundamental frequencies of rectangular composite plates.

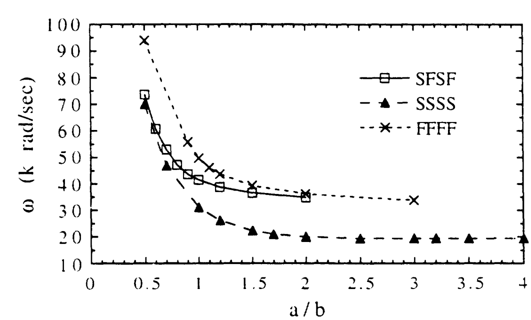
6. CONCLUSIONS

From the optimization analysis of freely vibrated laminated plates with various plate thicknesses, aspect ratios, circular cutouts and end conditions, the following conclusions may be drawn:

1. The end conditions has significant influence on the optimal fiber angles and optimal fundamental frequencies of rectangular and square laminated plates with and without cutouts.
2. When the aspect ratio becomes large, the optimal fiber angles for rectangular laminated plates with and without cutouts gradually approach to 90°, and the optimal fundamental frequencies attenuate to constant values.
3. The cutout has significant influence on the optimal fiber angles of thin and thick rectangular laminated plates with four simply supported ends and with two simply supported ends and two fixed ends. However, the cutout has no influence on the optimal fiber angles of plates with four fixed ends.
4. For square laminate thin plates, the optimal fiber orientations are insensitive to the sizes of cutouts for plates with four simply supported ends only. For square laminate thick plates, the optimal fiber angles are sensitive to the sizes of cutouts for plates with two simply supported ends and two fixed ends only.



(a) Aspect ratio a/b vs. optimal fiber angle θ



(b) Aspect ratio a/b vs. optimal fundamental natural frequency ω

Figure 10. ***Effect of end conditions and plate aspect ratios on optimal fiber angle and optimal fundamental natural frequency of $[\pm\theta/90/0]_{10s}$ rectangular composite plates with circular cutouts.***

5. The introduction of cutout may increase the optimal fundamental frequencies of rectangular laminate plates and the optimal fundamental frequencies of square laminate plates may increase with the increase of the sizes of cutouts.

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