provide a conservative design approach that does not impose excessive restrictions when high-strength concrete is used.

In response to the discussion prepared by Mark K. Johnson and Julio A. Ramirez, the authors acknowledge that a valid point has been raised regarding the recently adopted code provisions for shear strength of high-strength concrete members. Research conducted by the authors confirmed the applicability and conservativeness of the new code provisions for minimum required web reinforcement when the full value of  $\sqrt{f_c}$  is used in shear design. However, Johnson and Ramirez have pointed out that when designers elect to limit the value of  $\sqrt{f_c}$  used in shear design to 100 psi (690 kPa) and used minimum shear reinforcement of  $A_v = 50$   $b_w s/f_p$ , unconservative designs may result.

Test results from beam specimens 1 and 8 reported in the authors' research support the point raised by Johnson and Ramirez. These two beam specimens had a concrete compressive strength of approximately 18,000 psi (124 MPa) and fulfilled all the design requirements of ACI 318-89. The measured shear strengths  $V_{test}$  for these specimens are reported in Table 5 along with the calculated shear strength  $(V_n)$ . The reported  $V_n$  term was calculated based on ACI 318-83 code provision to

illustrate the lack of conservatism of the 1983 code provisions for high-strength concrete. However, if  $V_n$  is calculated using ACI 318-89 code provisions, limiting the  $\sqrt{f_c^{\prime}}$  to 100 psi (690 kPa) as required, the ratios of  $V_{test}/V_n$  are 0.89 and 0.80 for Specimens 1 and 8, respectively.

This point is further demonstrated through comparison of test results from beam specimens 6 and 8. These beam specimens had identical cross sections, identical quantities of shear reinforcement  $(A_v = 50 \ b_w s/f_v)$ , and similar reinforcment ratios. Beam Specimen 8 had a concrete compressive strength of approximately 18,000 psi (124 MPa) yet this beam failed in diagonal tension at a load that was less than the maximum load for Beam Specimen 6, which had a concrete compressive strength of approximately 10,000 psi (69 MPa). At this time, it is not clear why Beam Specimen 8 failed at a lower shear strength than Specimen 6. These test results do indicate, however, that the current code provisions, which conditionally limit the values of  $\sqrt{f_c'}$  used in shear design to 100 psi, may not be adequate. It appears that minimum shear reinforcement, as stipulated in Section 11.1.2.1 should be required whenever concrete with compressive strength in excess of 10,000 psi (69 MPa) is used.

Disc. 87-S21/From the March-April 1990 ACI Structural Journal, p. 199

# Nonlinear Analysis of Cracked Reinforced Concrete. Paper by Hsuan-Teh Hu and William C. Schnobrich

Discussion by P. D. Zararis and Authors

#### By P. D. ZARARIS

Member American Concrete Institute, Associate Professor of Civil Engineering, Aristotle University, Thessaloniki, Greece

The authors have presented a model for cracked reinforced concrete subjected to inplane shear and normal stresses. However, in their attempt to develop a model suitable for a finite element analysis, they have disregarded basic experimental findings while, on the contrary, they have emphasised nonrealistic concepts and considerations. This has repeatedly been the case with other authors presenting similar models.

First of all, the smeared crack representation is a fictitious but convenient device for a finite element analysis. However, such a representation that ignores the occurrence of individual cracks is not acceptable for a thorough treatment of the subject. Indeed, it is known that a decisive change in strains and stresses takes place at the location of the discrete cracks. The forces acting on these real cracks, and also the direction of the cracks, are of primary consideration. These quantities are necessary for the formation of the tensor of concrete stresses between the cracks, which determines the behavior of a structural element until collapse. <sup>16</sup>

Contrary to the previous statement, the authors state that a concern about crack localization is an unnecessary expense. Thus, instead of searching for the fixed direction of the cracks, the authors use a rotating crack approach and, while at one place in their paper the rotating cracks are defined as notational ones, in another place the change in the crack direction is mentioned as clearly observed in the experiments of Vecchio and Collins. But it is well known from the observation of every pattern of cracking, as well as from the analysis, 16 that cracks may appear in two directions at the most. Besides the original cracks, ultimate cracks have also formed in another direction in some of the previously mentioned experiments. This cannot possibly be interpreted as rotation of cracks. Furthermore, doubts have been raised as to whether the stress redistribution between the real cracks is such as to infer these imaginary rotating cracks.

In formulating the constitutive matrix for steel, the authors have assumed that the dowel action of the reinforcing steel is negligible and the bond between steel and concrete remains perfect. Both of the previous assumptions are incorrect. Regarding the first, there are shear forces of steel bars, caused by a transverse displacement (not slip) at the crack location, which play a very important role in the formation of the stress state of a structural element. Regarding the second, it is obvious that after the formation of dense discrete cracks and the simultaneous formation of microcracks around the steel bars, the bond is far from being per-

fect. On the contrary, it could be neglected, as the observation of steel strains between the cracks advocates.

The authors believe that a crack occurs when the concrete stresses reach the failure surface. No mention is made of the role that reinforcement plays. Nevertheless, it is well known that the presence of steel bars strongly affects both the cracking load and, mainly, the direction of cracks.

Regarding the stress degrading effect for concrete parallel to crack direction, the authors declare that experimental results show that the tensile cracks have caused damage to the concrete. However, this is not true. Experiments have shown nothing more than the undoubtedly existing degradation of concrete strength and stiffness. Concrete between the cracks is not altered into a new material. Its failure continues to follow precisely the well-known interaction diagram of concrete strength under biaxial stresses, as it was first given by Kupfer. What is necessary is to determine accurately both the tensile and compressive stresses of concrete between the cracks. These stresses have been found to be directly affected by the shear forces of steel bars. 16 As the yielded steel bars lose their shear forces, the authors' paper correctly states, as well as in Reference 2 of their paper, that when the reinforcing steel yields, the average tensile stress of cracked concrete is close to zero.

In the comparison with experimental results, the authors have used for their analytical results a tensile strength of concrete  $(f'_t = 0.33 \sqrt{f'_c})$ , which is far too low compared to a real one  $(f'_t = 0.3 f_c^{c2/3})$ . This degraded tensile strength perhaps suits the diagrams used by the authors, but it gives incorrect values (lower ones) of concrete strength when the real interaction diagram of biaxial stresses is used.

Finally, the existence of the failure mode  $C_1$  presented by the authors, i.e., concrete failure after the yielding of reinforcement only in one direction, is disputable. After the yielding of steel in one direction, the transverse tensile stresses of the concrete are considerably reduced, and the strength of the concrete in biaxial stress conditions becomes higher. In this case, the failure might be attributed to a great shear deformation (like a slip), as is evident in the experiments. But this shear deformation mostly takes place in the direction of ultimate cracks (PV19, PV20), i.e., it occurs at the yielding of steel in the other direction, too. Thus, this failure mode may be regarded as identical with that of yielding of steel in both directions.

## REFERENCE

16. Zararis, Prodromos D., "Failure Mechanisms in R/C Plates Carrying In-Plane Forces," Journal of Structural Engineering, ASCE, V. 114, No. 3, Mar. 1988, pp. 553-574.

#### **AUTHORS' CLOSURE**

The authors thank Professor Zararis for his comments, which have been carefully reviewed and are answered in the following paragraphs.

For cases in which a microscopic point of view is necessary, the authors agree with Professor Zararis that

a decisive change in strains and stresses takes place at the location of well-developed discrete cracks. However, on a macroscopic level, if overall load-deflection behavior is desired, without regard to precisely computed crack patterns and local stresses, the smeared crack representation is a viable and perhaps a best choice. 6,17,18 In general, overall structural behavior of some large complex systems requires only that the overall stiffnesses of the various constituents reflect the average conditions of the various regions. Problems such as the determination of the strength of cooling towers under wind loading, the collapse mechanism of hypar roof with edge beams, etc., can indeed be studied with a simple smeared crack approach with its changing material constitutive matrix. The use of this smeared crack concept has proven to be quite satisfactory for a number of reinforced concrete structure studies.9,12,15,19-21

In anisotropically reinforced concrete elements which have different amounts of reinforcement in two orthogonal directions, the internal resisting forces do not remain proportional upon cracking. With isotropically reinforced concrete elements, if the steel in one direction yields first, the proportionality of internal resisting forces is also lost. Therefore in cases such as these the principal stress and strain directions do not remain fixed anymore and the crack directions start to shift. The change in the crack direction and the consequential change in direction of the maximum stiffness has been clearly observed in the experiments of Vecchio and Collins.2 In the finite element anlaysis of reinforced concrete, a conventional fixed crack model cannot handle this crack shift effect, and the need for a rotating crack model becomes quite apparent.4,15

Bond between concrete and steel is important to most aspects of localized reinforced concrete behavior. In the authors' paper, though perfect bond between concrete and steel reinforcement is assumed, the effect of the bond stresses is already represented by the inclusion of tension stiffening in the concrete between the cracks.

The post-cracking shear resistance mechanisms due to aggregate interlock or shear friction and dowel action are significant contributors in many types of reinforced concrete members and have attracted considerable research effort in recent years.6 Though the dowel action of the reinforcing steel is assumed to be neglected in the authors' paper, the entire shear transfer mechansims due to aggregate interlock and dowel action have been incorporated indirectly into the constitutive matrix of cracked concrete through the shear retention factor  $\mu$ . Actual inclusion of dowel action introduces the need to consider crack opening and flexural stiffness, deformation, etc. This means the local damage the reinforcing steel does at the crack face needs to be evaluated. Computations can become lengthy. The concrete-steel interaction problem is complex, as evident in Chapter 3 of the ASCE committee report.6

The authors agree that the presence of steel bars strongly affects both the cracking load and the direction of cracks. Definitely, the cracking load as well as the direction of cracks are different for plain concrete and reinforced concrete under the same loading environment. In the authors' model, the role that reinforcement plays has been incorporated into the constitutive matrix of steel, which has been further assembled into the constitutive matrix of the entire reinforced concrete structure.

Regarding the stress degrading effect in the concrete parallel to the crack direction, the authors quite agree with Professor Zararis that "concrete between the cracks is not altered into a new material. Its failure continues to follow precisely the well known interaction diagram of concrete under biaxial stress." Professor Zararis suggests that, in order to calculate the tensile and degraded compressive stress of concrete between cracks accurately, the shear force of the steel bar must be considered. In the authors' paper, the tensile strain normal to the crack direction is used as a parameter to determine the degraded compressive stress and stiffness for concrete parallel to crack direction, while the shear force of the steel bar is included in the constitutive matrix of cracked concrete through the shear retention factor  $\mu$ , as mentioned in the previous paragraph. It is the authors' opinion that though these two methods are different approaching the problem, both methods are appropriated in anlayzing the global behavior of cracked reinforced concrete elements.

As to the issue of tensile strength of concrete, Mr. Zararis suggests using a value of  $f'_t = 0.3f_c^{2/3}$ . However, in view of the scatter in the cracking stresses obtained by Vecchio and Collins,<sup>2</sup> it was decided to use  $f'_t = 0.33\sqrt{f'_c}$  MPa in its traditional and conservative form.

Finally, Prof. Zararis suggestst that the existence of the failure mode  $C_1$ , i.e., concrete failure after yielding

of the weaker reinforcement but prior to yielding of the stronger reinforcement, may be regraded as identical with that of yielding of steel in both directions. However, panels with the yielding of steel in both directions (PV11, PV16) could still maintain the same ultimate loading after the yield of the steel in the second direction and displayed very substantial deformations prior to failure. On the other hand, while panels that failed in the  $C_1$  mode (PV19, PV20) displayed considerable ductility immediately after the yield of weaker steel, these panels could still resist increasing load. In numerical analysis, the stronger steel of these panels never yielded. The authors' think these two failure mechanisms should be made different.

#### **REFERENCES**

- 17. Chen, Wai-Fah, Plasticity in Reinforced Concrete, McGraw-Hill Book Co., New York, 1982, 474 pp.
- 18. Schnobrich, W. C., "The Role of Finite Element Analysis of Reinforced Concrete Structures," *Proceedings*, Seminar on Finite Element Analysis of Reinforced Concrete Structures, Japan Society for the Promotion of Science/U.S. National Science Foundation, 1986, pp. 1-24.
- 19. Lin, Cheng-Shung, and Scordelis, Alexander C., "Nonlinear Analysis of RC Shells of General Form," *Proceedings*, ASCE, V. 101, ST3, Mar. 1975, pp. 523-538.
- 20. Hand, F. R.; Pecknold, D. A.; and Schnobrich, W. C., "A Layered Finite Element Nonlinear Analysis of Reinforced Concrete Plates and Shells," Civil Engineering Studies, *Structural Research Series* No. 389, University of Illinois, Urbana, Aug. 1972.
- 21. Chan, E. C., "Nonlinear Geometric, Material and Time Dependent Analysis of Reinforced Concrete Shells with Edge Beams," PhD thesis, UC-SESM 82-8, University of California, Berkeley, Aug.

Disc. 87-S22/From the March/April 1990 ACI Structural Journal, p. 208

#### Punching Shear Strength in Concrete Slabs, Paper by Lionello Bortolotti

Late discussion has been received; publication has been postponed to the March-April 1991 ACI Structural Journal to allow the author to prepare a closure.

Disc. 87-S23/From the March-April 1990 ACI Structural Journal, p. 220

Influence of Corrosion and Cracking on Bond Behavior and Strength of Reinforced Concrete Members, Paper by G. J. Al-Sulaimani, M. Kaleemullah, I. A. Basunbul, and Rasheeduzzafar

Discussion by J. G. Cabrera and P. Ghoddoussi

### By J. G. CABRERA and P. GHODDOUSSI

Member American Concrete Institute, Department of Civil Engineering, University of Leeds, UK; and Department of Civil Engineering, University of Leeds

The authors have presented a very interesting paper; however, there are some aspects of the experimental work which should be made clear. These are:

ACI Structural Journal / January-February 1991

- 1. There is no mention of the manner in which corrosion was induced. It has to be assumed that the specimens for pullout tests were immersed in a salt solution and that the beam specimens were carbonated. What was the extent of carbonation and what was the salt used?
- b. Specimens were subjected to a constant current density of 2mA/cm<sup>2</sup> (20 A/m<sup>2</sup>). The duration of ap-