Buckling Optimization of Fiber-Composite Laminate Shells with Large Deformation

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ABSTRACT

The buckling strength of fiber-composite laminate shells with a given material system is maximized with respect to fiber orientations. While a modified Riks nonlinear incremental algorithm is utilized to calculate the buckling load and to study the postbuckling behavior of the composite shells, a sequential linear programming method together with a simple move limit strategy is used to optimize the buckling strength of the shells. Results of this optimization study for simply supported composite cylindrical shells subjected to external hydrostatic compression and with different laminate layups, $[\pm\theta/90_2/0]_s$ and $[\theta/\phi/90_2/0]_s$, are presented.

INTRODUCTION

Applications of fiber composite materials (Fig. 1) to advanced shell structures such as aircraft fuselages, deep submersibles and surface ships have been increased rapidly in recent years. These composite laminate shells in service are commonly subjected to various kinds of external loading which may induce buckling. In many situations buckling is an undesirable phenomenon. Hence, structural instability becomes a major concern in safe and reliable design of the advanced composite shells. The buckling strength of fiber composite shells heavily depends on ply orientations, e.g. Sun and Hansen [1], Hu and Wang [2]. Therefore, the proper selection of appropriate fiber orientations for a given composite material system to achieve the maximum buckling strength of composite shells becomes a crucial problem.

Researches on the subject of structural optimization have been reported by many investigators, e.g. Schmit [3]. Among various optimization schemes, the sequential linear programming method is one of the most popular approaches in solving structural optimization problems and it has been successfully applied to many large scale structural problems, e.g. Vanderplaats [4], Haftka et al. [5], Zienkiewicz and Champbell [6]. Hence, in this paper the sequential linear programming method has been adopted and used together with a simple move-limit strategy to optimize the buckling strength of fiber-composite laminate cylindrical shells with respect to the fiber orientations.

This study uses a modified Riks nonlinear incremental algorithm implemented in the ABAQUS finite element program [7] to calculate the buckling loads and to study the postbuckling behavior of composite shells. For the purpose of comparison, optimization based on linearized buckling analysis is also carried out. In this paper, first the linearized buckling analysis, the nonlinear buckling analysis, the constitutive matrix formulation for composite shells, and the sequential linear programming method are briefly discussed. Then the results of the buckling optimization for simply supported composite cylindrical shells subjected to external hydrostatic compression and with different laminate layups, $[\pm\theta/90_2/0]_s$ and $[\theta/\phi/90_2/0]_s$, are presented. Finally, conclusions obtained from the study are given.

LINEARIZED BUCKLING ANALYSIS

In a finite-element modeling scheme for nonlinear problems, the load-displacement relationship for a structure can be expressed in an incremental form as follows:

$$[K]_t\, d\{U\} = d\{P\} \tag{1}$$

where $[K]_t$ is a tangent stiffness matrix, $d\{U\}$ an incremental nodal displacement vector and $d\{P\}$ an incremental nodal force vector.

If it is assumed that the linear theory of small deformation before buckling holds, the linearized buckling formulation, e.g. Cook et al. [8], then leads to a tangent stiffness matrix with the following expression:

$$[K]_t = [K]_L + [K]_\sigma \tag{2}$$

where $[K]_L$ is a linear stiffness matrix and $[K]_\sigma$ a geometric stiffness matrix dependent upon stresses.

The bifurcation solution for the linearized buckling problem then may be determined from the following eigenvalue equation:

$$([K]_L + \lambda[K]_{\sigma_0})\,\{\psi\} = \{0\} \tag{3}$$

where λ is an eigenvalue and $\{\psi\}$ an eigenvector. The critical buckling load P_{cr} can be found from $P_{cr} = \lambda P_0$ where P_0 is the nominal load which corresponds to the stress state σ_0.

NONLINEAR BUCKLING ANALYSIS

In the finite element program ABAQUS, the nonlinear response of structures is modeled through the incremental updated Lagrangian formulation, e.g. Bathe [9]. In order to model the potential decrease in loads and displacements as the solution evolves, a modified Riks nonlinear incremental algorithm [7] in ABAQUS is used to construct the equilibrium solution path. In this algorithm, the nonlinear procedure is based on a motion of a given distance along the tangent line (defined by the tangent stiffness matrix) to the current solution point. Then search for an equilibrium solution in the plane, that passes through the current solution point and that is orthogonal to the same tangent line, can be carried out using an iterative algorithm.

In order to model bifurcation from the prebuckling path to the postbuckling path, a geometric imperfection of the composite shell can be introduced by superimposing a small fraction of the lowest eigenmode, which is determined by a linearized buckling analysis, to the original nodal coordinates of the shell as follows:

$$\{I\} = \{O\} + \varepsilon t\{\psi\} \tag{4}$$

where $\{I\}$ is the resulting imperfect nodal coordinates of the shell, $\{O\}$ is the original nodal coordinates of the shell, ε is a scaling coefficient, t is the thickness of the shell, and $\{\psi\}$ is the normalized lowest eigenmode. In this study, ε is taken to be 0.001 for all the nonlinear buckling analyses.

CONSTITUTIVE MATRIX FOR FIBER-COMPOSITE LAMINAE

The elements used in the numerical analyses are eight-node isoparametric shell elements with six degrees of freedom per node (three displacements and three rotations). The shell formulation is based on Mindlin-type displacement field assumptions which allows transverse shear deformation [7].

During a finite element analysis, the constitutive matrices of composite materials at element integration points must be calculated before the stiffness matrices are assembled from element level to structural level. For fiber-composite laminate materials, each lamina can be considered as an orthotropic layer in a plane stress condition. The stress-strain relations for an orthotropic lamina in the material coordinates (Fig. 1) at an integration point can be written as

$$\{\sigma'\} = [Q_1']\,\{\varepsilon'\} \tag{5}$$

$$\{\tau_t'\} = [Q_2']\,\{\gamma_t'\} \tag{6}$$

$$[Q_1'] = \begin{bmatrix} \dfrac{E_{11}}{1-\nu_{12}\nu_{21}} & \dfrac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ \dfrac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} & \dfrac{E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}, \quad [Q_2'] = \begin{bmatrix} \alpha_1 G_{13} & 0 \\ 0 & \alpha_2 G_{23} \end{bmatrix} \tag{7}$$

where $\{\sigma'\} = \{\sigma_1, \sigma_2, \tau_{12}\}^T$, $\{\tau_t'\} = \{\tau_{13}, \tau_{23}\}^T$, $\{\varepsilon'\} = \{\varepsilon_1, \varepsilon_2, \gamma_{12}\}^T$, $\{\gamma_t'\} = \{\gamma_{13}, \gamma_{23}\}^T$. The α_1 and α_2 are shear correction factors and are taken to be 0.83 in this study. The constitutive equations for the lamina in the element coordinates become

$$\{\sigma\} = [Q_1]\{\varepsilon\}, \qquad [Q_1] = [T_1]^T [Q_1'] [T_1] \tag{8}$$

$$\{\tau_t\} = [Q_2]\{\gamma_t\}, \qquad [Q_2] = [T_2]^T [Q_2'] [T_2] \tag{9}$$

$$[T_1] = \begin{bmatrix} \cos^2\phi & \sin^2\phi & \sin\phi\cos\phi \\ \sin^2\phi & \cos^2\phi & -\sin\phi\cos\phi \\ -2\sin\phi\cos\phi & 2\sin\phi\cos\phi & \cos^2\phi-\sin^2\phi \end{bmatrix} \tag{10}$$

$$[T_2] = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \tag{11}$$

where $\{\sigma\} = \{\sigma_x, \sigma_y, \tau_{xy}\}^T$, $\{\tau_t\} = \{\tau_{xz}, \tau_{yz}\}^T$, $\{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \gamma_{xy}\}^T$, $\{\gamma_t\} = \{\gamma_{xz}, \gamma_{yz}\}^T$, and ϕ is measured counterclockwise from the element local x-axis to the material 1-axis. Assume $\{\varepsilon_o\} = \{\varepsilon_{xo}, \varepsilon_{yo}, \gamma_{xyo}\}^T$ are the in-plane strains at the mid-surface of the section and $\{\kappa\} = \{\kappa_x, \kappa_y, \kappa_{xy}\}^T$ are the curvatures. The in-plane strains at a distance, z, from the mid-surface become:

$$\{\varepsilon\} = \{\varepsilon_o\} + z\{\kappa\} \tag{12}$$

If h is the total thickness of the section, the stress resultants, $\{N\} = \{N_x, N_y, N_{xy}\}^T$, $\{M\} = \{M_x, M_y, M_{xy}\}^T$ and $\{V\} = \{V_x, V_y\}^T$, can be defined as

$$\{N\} = \int_{-h/2}^{h/2} \{\sigma\}dz = \int_{-h/2}^{h/2} [Q_1](\{\varepsilon_o\}+z\{\kappa\})dz \tag{13.a}$$

$$\{M\} = \int_{-h/2}^{h/2} z\{\sigma\}dz = \int_{-h/2}^{h/2} z[Q_1](\{\varepsilon_o\}+z\{\kappa\})dz \tag{13.b}$$

$$\{V\} = \int_{-h/2}^{h/2} \{\tau_t\}dz = \int_{-h/2}^{h/2} [Q_2]\{\gamma_t\}dz \tag{13.c}$$

If there are n layers in the layup, the constitutive matrix for composite materials at an element integration point can be written as a summation of integrals over the n laminae in the following form:

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{V\} \end{Bmatrix} = \sum_{j=1}^{n} \begin{bmatrix} (z_{jt}-z_{jb})[Q_1] & \frac{1}{2}(z_{jt}^2-z_{jb}^2)[Q_1] & [0] \\ \frac{1}{2}(z_{jt}^2-z_{jb}^2)[Q_1] & \frac{1}{3}(z_{jt}^3-z_{jb}^3)[Q_1] & [0] \\ [0]^T & [0]^T & (z_{jt}-z_{jb})[Q_2] \end{bmatrix} \begin{Bmatrix} \{\varepsilon_o\} \\ \{\kappa\} \\ \{\gamma_t\} \end{Bmatrix} \tag{14}$$

where z_{jt} and z_{jb} are the distance from the mid-surface of the section to the top and the bottom of the j-th layer respectively. The [O] is a 3 by 2 matrix with all the coefficients equal to zero.

SEQUENTIAL LINEAR PROGRAMMING

A general optimization problem may be defined as the following:

Minimize: $f(\underline{x})$ (15.a)

Subjected to: $g_i(\underline{x}) \le 0, \quad i = 1, ..., r$ (15.b)

$h_j(\underline{x}) = 0, \quad j = r+1, ..., m$ (15.c)

$p_k \le x_k \le q_k, \quad k = 1, ..., n$ (15.d)

where $f(\underline{x})$ is an objective function, $g_i(\underline{x})$ are inequality constraints, $h_j(\underline{x})$ are equality constraints, and $\underline{x} = \{x_1, x_2, ..., x_n\}^T$ is a vector of design variables. If a particular optimization problem requires maximization, we simply minimize $-f(\underline{x})$.

The concept of a sequential linear programming is that given a feasible solution $\underline{x}_o = \{x_{o1}, x_{o2}, ..., x_{on}\}^T$ for an optimization problem, a linear programming problem may be established by expanding the nonlinear functions about $\underline{x}_o$ in a Taylor series, and ignoring terms of order higher than the linear ones. With this approximation, the optimization problem, Eqs. (15.a-15.d), becomes:

Minimize: $f(\underline{x}) \approx f(\underline{x}_o) + \nabla f(\underline{x}_o)^T \delta\underline{x}$ (16.a)

Subjected to: $g_i(\underline{x}) \approx g_i(\underline{x}_o) + \nabla g_i(\underline{x}_o)^T \delta\underline{x} \le 0$ (16.b)

$h_j(\underline{x}) \approx h_j(\underline{x}_o) + \nabla h_j(\underline{x}_o)^T \delta\underline{x} = 0$ (16.c)

$p_k \le x_k \le q_k$ (16.d)

where $\delta\underline{x} = \{x_1-x_{o1}, x_2-x_{o2}, ..., x_n-x_{on}\}^T$. A solution for Eqs. (16.a-16.d) may be easily obtained by the simplex method, e.g. Kolman and Beck [10]. After obtaining an initial approximate solution for Eqs. (16.a-16.d), say $\underline{x}_1$, we can linearize the original problem, Eqs. (15.a-15.d), at $\underline{x}_1$ and solve the new linear programming problem. The process is repeated until a convergent solution is obtained.

Although the procedure for a sequential linear programming is simple, difficulties may arise during the iterations. First, the optimum solution for the approximate linear problem may violate the constraint conditions of the original optimization problem. Second, in a nonlinear problem, the true optimum solution may appear between two constraint intersections. A straightforward successive linearization in such a case may lead to an oscillation of the solution between the widely separated values. Difficulties in dealing with such problems may be avoided by imposing a "move limit" on the linear approximation, e.g. Vanderplaats [4], Haftka et al. [5], Zienkiewicz and Champbell [6]. The concept of a move limit is that a set of box-like admissible constraints are placed in the range of $\delta\underline{x}$. In general, the choice of a suitable move limit depends on experience and also on the results of previous steps. Once a proper move limit is chosen at the beginning of the sequential linear programming procedure, this move limit should gradually approach to zero as the iterative process continues, e.g. Vanderplaats [4], Zienkiewicz and Champbell [6], Esping [11].

The algorithm of a sequential linear programming with selected move limits may be summarized as follows: (1) Linearize the nonlinear objective function and associated constraints with respect to an initial guess $\underline{x}_0$. (2) Impose move limits in the form of $-\underline{S} \le (\underline{x} - \underline{x}_0) \le \underline{R}$, where $\underline{S}$ and $\underline{R}$ are properly chosen positive constraints. (3) Solve the approximate linear programming problem to obtain an initial optimum solution $\underline{x}_1$. (4) Repeat the process by redefining $\underline{x}_1$ with $\underline{x}_0$ until either the subsequent solutions do not change significantly (i.e., true convergence) or the move limit approaches to zero (i.e., forced convergence).

NUMERICAL ANALYSIS

Buckling Optimization of Composite Shell with One Design Variable

In this section, a simply supported fiber-composite laminate cylindrical shell (Fig. 2) with laminate layup $[\pm\theta/90_2/0]_s$ under external hydrostatic compression is investigated. The objective of this study is to determine the optimal fiber angle θ to maximize the buckling load p_{cr} of the shell and to compare the result of the optimization using nonlinear buckling analysis with that using linearized buckling analysis.

Based on the sequential linear programming method, in each iteration the current, linearized optimization problem becomes:

Maximize: $$p_{cr}(\theta) \approx p_{cr}(\theta_0) + (\theta-\theta_0)\left.\frac{\partial p_{cr}}{\partial \theta}\right|_{\theta=\theta_0} \qquad (17.a)$$

Subjected to: $$0^o \le \theta \le 90^o \qquad (17.b)$$

$$-r \times q \times 0.5^s \le (\theta-\theta_0) \le r \times q \times 0.5^s \qquad (17.c)$$

where θ_0 is a solution in the current iteration. The r and q are the size and the reduction rate of the move limit. In this study, the values of r and q are selected to be 10^o and $0.9^{(N-1)}$, where N is a current iteration number. To control the oscillation of the solution, a parameter 0.5^s is introduced in the move limit, where s is the number of oscillation of the derivative $\partial p_{cr}/\partial\theta$ that has taken place before the current iteration. The value of s increases by 1 if the sign of $\partial p_{cr}/\partial\theta$ changes. Whenever oscillation of the solution occurs, the range of the move limit is reduced to half of its current value, which is similar to a bisection method, e.g. Maror [12]. This expedites the solution convergent rate very rapidly.

The derivative $\partial p_{cr}/\partial\theta$ in Eq. (17.a) may be approximated by using a forward finite-difference method as follows:

$$\frac{\partial p_{cr}}{\partial \theta} \approx \frac{[p_{cr}(\theta_0+\Delta\theta)-p_{cr}(\theta_0)]}{\Delta\theta} \qquad (18)$$

In order to determine the value of $\partial p_{cr}/\partial\theta$ in Eq. (18), two buckling analyses are needed to compute $p_{cr}(\theta_0)$ and $p_{cr}(\theta_0+\Delta\theta)$ in each iteration. In this study, the value of $\Delta\theta$ is selected to be 1^o in most iterations.

Important numerical results obtained in optimization study are given in Fig. 3, which shows the fiber orientation θ and the associated critical buckling pressure p_{cr} determined in each iteration for the shell. The initial values of θ are selected to be 90^o for linearized buckling analysis as well as nonlinear buckling analysis. Both solutions converged within 12 iterations. Though, the optimal critical buckling pressure, 24.7 ksi, computed by using linearized buckling analysis is lower than that, 25.5 ksi, computed by using nonlinear buckling analysis, the optimal value of θ converges to 60.8^o for both analyses.

For the optimization study using the nonlinear buckling analysis, in each iteration two nonlinear buckling analyses to calculate the derivative information of Eq. (18) and two linearized buckling analyses to generate the initial imperfections for the nonlinear analyses are required. Since the trends in finding the optimal fiber orientation are the same using both buckling analyses, it is suggested that the calculation of the derivative using the nonlinear buckling analysis may be substituted by that using the linearized buckling analysis. The result is that in each iteration only one nonlinear buckling analysis to evaluate the critical buckling pressure and two linearized buckling analyses to compute the derivative are needed. The elimination of one massive nonlinear buckling analysis will significantly reduce the computer time for the entire optimization calculation.

Figure 4 shows the load-end displacement curves for the composite shell associated with the first iteration and the final iteration (optimal solution), which are computed by using the nonlinear buckling analysis. It can be seen that under the optimal condition, not only the critical buckling pressure of the shell is increased but also the post buckling strength of the shell is greatly improved.

Buckling Optimization of Composite Shell with Two Design Variables

In this section, the composite laminate shell with the same geometry, end conditions and loading conditions as that in the previous section but with laminate layup $[\theta/\phi/90_2/0]_s$ is investigated. Here, the constraint on the fiber angle $\pm\theta$ set in the previous section has been released. The objective of this study is then to find the optimal fiber orientations θ and ϕ, and to examine how the change of the fiber angle constraint will influence the optimal critical buckling pressure p_{cr}.

Based on the sequential linear programming method, in each iteration the current, linearized optimization problem becomes:

Maximize: $$p_{cr}(\theta,\phi) = p_{cr}(\theta_o,\phi_o) + (\theta-\theta_o)\frac{\partial p_{cr}}{\partial \theta}\bigg|_{\theta=\theta_o,\phi=\phi_o} + (\phi-\phi_o)\frac{\partial p_{cr}}{\partial \phi}\bigg|_{\theta=\theta_o,\phi=\phi_o} \quad (19.a)$$

Subjected to:
$$0^o \leq \theta \leq 90^o \quad (19.b)$$
$$-90^o \leq \phi \leq 0^o \quad (19.c)$$
$$-r_1 \times q_1 \times 0.5^{s_1} \leq (\theta-\theta_o) \leq r_1 \times q_1 \times 0.5^{s_1} \quad (19.d)$$
$$-r_2 \times q_2 \times 0.5^{s_2} \leq (\phi-\phi_o) \leq r_2 \times q_2 \times 0.5^{s_2} \quad (19.e)$$

where θ_o and ϕ_o are the solutions in the current iteration. The values of r_1 and r_2 (the sizes of move limits) are selected to be 10^o. The values of q_1 and q_2 (the reduction rates of move limits) are selected to be $0.9^{(N-1)}$, where N is a current iteration number. The s_1 and s_2 are the number of oscillation of the derivatives $\partial p_{cr}/\partial\theta$ and $\partial p_{cr}/\partial\phi$. The derivative terms in Eq. (19.a) may be approximated with the following finite-difference forms:

$$\frac{\partial p_{cr}}{\partial \theta} \approx \frac{[p_{cr}(\theta_o+\Delta\theta,\phi_o)-p_{cr}(\theta_o,\phi_o)]}{\Delta\theta} \quad (20.a)$$

$$\frac{\partial p_{cr}}{\partial \phi} \approx \frac{[p_{cr}(\theta_o,\phi_o+\Delta\phi)-p_{cr}(\theta_o,\phi_o)]}{\Delta\phi} \quad (20.b)$$

In this optimization study, three linearized buckling analyses are used to calculate the derivative information in Eqs. (20.a) and (20.b), and one nonlinear buckling analysis is used to evaluate p_{cr} in Eq. (19.a). The values of $\Delta\theta$ and $\Delta\phi$ are selected to be 1^o in most iterations.

Important numerical results obtained in optimization study are given in Fig. 5, which shows the fiber orientations θ and ϕ, and the associated critical buckling pressure p_{cr} determined in each iteration for the shell. The initial values of θ and ϕ are selected to be 90^o and -90^o. After twelve iterations, the optimal values of θ and ϕ converge to 63.4^o and -59.0^o respectively and the optimal critical buckling pressure converges to 25.6 ksi. Figure 6 shows the load-end displacement curves for the composite shell associated with the first iteration and the final iteration (optimal solution). Again, it can be seen that under the optimal condition, not only the critical buckling pressure of the shell is increased but also the post buckling strength of the shell is greatly improved.

Comparing the optimal critical buckling pressures p_{cr}, 25.6 ksi, obtained from this optimization analysis with that, 25.5 ksi, obtained

from previous optimization analysis, one can observe that the increase in the optimal critical buckling pressure is very insignificant. Figure 7 shows the load-end displacement curves under optimal fiber angle conditions for the composite shell with $[\pm\theta/90_2/0]_s$ laminate layup and $[\theta/\phi/90_2/0]_s$ laminate layup. It can be seen that the postbuckling strengths are also almost the same for the shell with these two different laminate layups. Therefore, it can be concluded that the optimal buckling behavior of the composite shell with $[\pm\theta/90_2/0]_s$ laminate layup is about the same as that of the composite shell with $[\theta/\phi/90_2/0]_s$ laminate layup. Hence, the optimization of the composite shell using two design variables may be undesirable since it costs more computation time.

CONCLUSIONS

From the optimization results obtained in this study, the following conclusions can be drawn:

1. For the optimization of a simply supported $[\pm\theta/90_2/0]_s$ composite shell under external hydrostatic compression using the sequential linear programming formulation, the trends in finding the optimal fiber orientation are the same for using linearized buckling analyses and for using nonlinear buckling analysis. Hence, the calculation of the derivative information using the nonlinear buckling analysis may be substituted by that using the linearized buckling analysis.
2. The optimal buckling behavior of the simply supported $[\pm\theta/90_2/0]_s$ composite shell under external hydrostatic compression is about the same as that of the composite shell with $[\theta/\phi/90_2/0]_s$ laminate layup. Therefore, the optimization of the composite shell using two design variables may be undesirable since it costs more computation time.

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REFERENCES

1. Sun, G. and Hansen, J. S. Optimal Design of Laminated-Composite Circular-Cylindrical Shells Subjected to Combined Loads, Journal of Applied Mechanics, ASME, Vol. 55, pp. 136-142, 1988.
2. Hu, H.-T. and Wang, S. S. Optimization for Buckling Resistance of Fiber-Composite Laminate Shells with and without Cutouts, pp. 1300-1312, Proceedings of the 31st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Long Beach, California, 1990.
3. Schmit, L. A. Structural Synthesis - Its Genesis and Development, AIAA Journal, Vol. 19, pp. 1249-1263, 1981.

4. Vanderplaats, G. N. Numerical Optimization Techniques for Engineering Design with Applications, McGraw-Hill, New York, 1984.

5. Haftka, R. T., Gurdal, Z. and Kamat, M. P. Elements of Structural Optimization, Second Revised Edition, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1990.

6. Zienkiewicz, O. C. and Champbell, J. S. Shape Optimization and Sequential Linear Programming, Chapter 6, Optimum Structural Design (Ed. Gallagher, R. H. and Zienkiewicz, O. C.), pp. 109-126, Wiley, New York, 1973.

7. Hibbitt, Karlsson & Sorensen, Inc. ABAQUS Theory Manual and User Manual, Version 4-8, Providence, Rhode Island, 1990.

8. Cook, R. D., Malkus, D. S. and Plesha, M. E. Concepts and Applications of Finite Element Analysis, Third Edition, Chapter 14, John Wiley & Sons, Inc., New York, 1989.

9 Bathe, K.-J. Finite Element Procedures in Engineering Analysis, Chapter 6, Prentice-Hall, New Jersey, 1982.

10. Kolman, B. and Beck, R. E. Elementary Linear Programming with Applications, Chapter 2, Academic Press, Orlando, 1980.

11. Esping, B. J. D. Minimum Weight Design of Membrane Structures, Computers and Structures, Vol. 19, pp. 707-716, 1984.

12. Maror, M. J. Numerical Analysis, Second Edition, Chapter 2, Macmillan Publishing Company, New York, 1987.

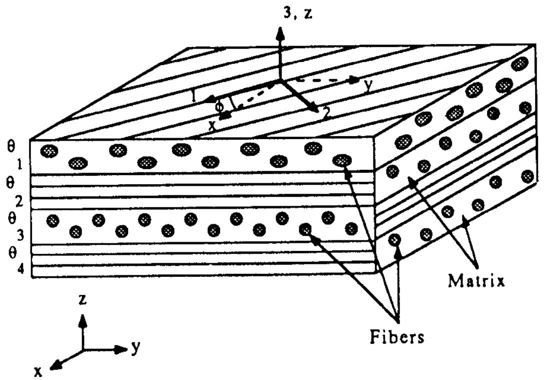
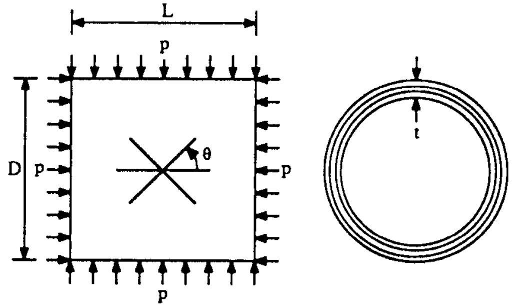


Figure 1 Material and element coordinate systems for fiber composite laminate

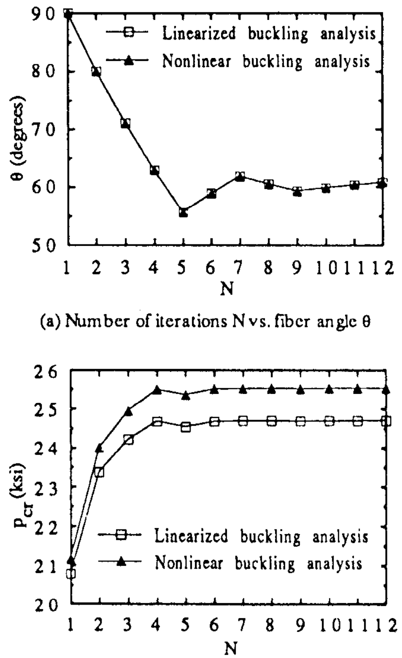


Laminate layup:
$[\pm\theta/90_2/0]_s$

Composite shell geometry:
L = 8 in.
D = 8 in.
t = 0.5 in.

Ply constitutive properties:
E_{11} = 20 Msi
E_{22} = 2.1 Msi
$G_{12} = G_{13}$ = 0.85 Msi
G_{23} = 0.51 Msi
ν_{12} = 0.21

Figure 2 Cylindrical composite laminate shell under external hydrostatic compression



(a) Number of iterations N vs. fiber angle θ

(b) Number of iterations N vs. critical pressure p_{cr}

Figure 3 Buckling optimization of simply supported $[\pm\theta/90_2/0]_s$ composite shell under hydrostatic compression, (a) number of iterations N vs. fiber angle θ, and (b) number of iterations N vs. critical pressure p_{cr}

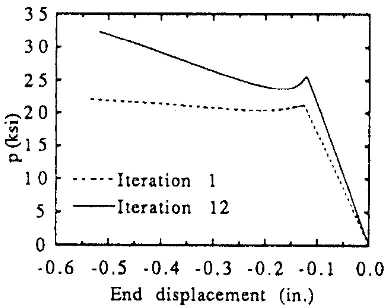
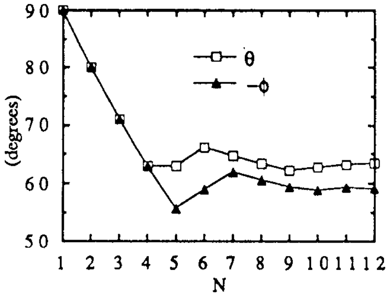
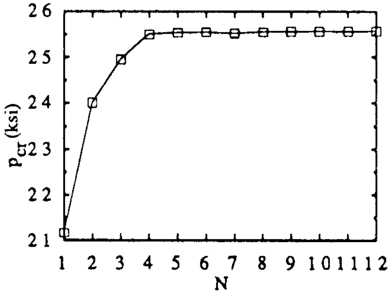


Figure 4 Load-end displacement curves for simply supported $[\pm\theta/90_2/0]_s$ composite shell under hydrostatic compression



(a) Number of iterations N vs. fiber angles θ and ϕ



(b) Number of iterations N vs. critical pressure p_{cr}

Figure 5 Buckling optimization of simply supported $[\theta/\phi/90_2/0]_s$ composite shell under hydrostatic compression, (a) number of iterations N vs. fiber angles θ and ϕ, and (b) number of iterations N vs. critical pressure p_{cr}

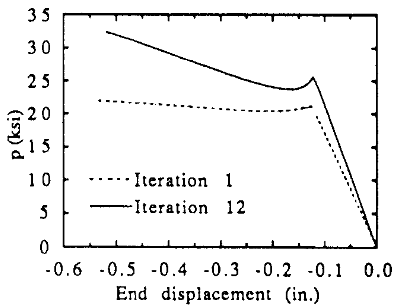


Figure 6 Load-end displacement curves for simply supported $[\theta/\phi/90_2/0]_s$ composite shell under hydrostatic compression

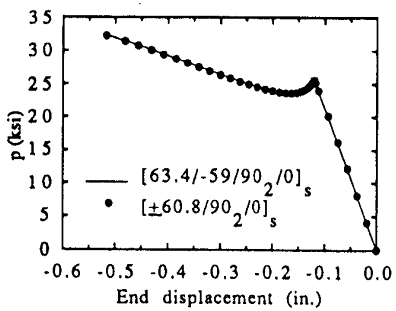


Figure 7 Load-end displacement curves for simply supported composite shell with $[63.4/-59/90_2/0]_s$ and $[\pm 60.8/90_2/0]_s$ laminate layups